Educational Technology Quarterly, Vol. **2025**, Iss. 1, pp. 42–66 https://doi.org/10.55056/etq.858

Evolution of digital competence development for mathematics educators in technical universities: a contemporary framework

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Abstract. The continuing digital transformation of higher education necessitates the ongoing development of educators' digital competencies, particularly in mathematics education within technical universities. This paper revisits and modernizes the conceptual framework for digital competence development among mathematics educators in light of recent technological advances and the accelerated digitalization prompted by global educational disruptions. Building upon previous work, we present an updated model of digital competence that incorporates contemporary frameworks such as DigCompEdu and the latest UNESCO ICT Competency Framework. The paper analyzes the evolving technological landscape in mathematics education, including the integration of cloud-based tools, artificial intelligence, and immersive technologies. Additionally, we propose a revised curriculum for professional development that addresses current needs and emerging trends. The multilevel competency development model presented accounts for varied starting points in digital literacy and provides a pathway for sustainable professional growth. Recommendations are provided to support the digital transformation of mathematics education in technical universities, with connections to broader educational quality goals.

Keywords: digital competence, mathematics education, higher education, technical universities, professional development, ICT integration, digital transformation, multilevel competency model, teacher education, educational technology, cloud-based mathematics tools, artificial intelligence in education, learning analytics, TPACK framework, DigCompEdu

1. Introduction

The digital transformation of higher education continues to reshape teaching and learning paradigms across disciplines. Mathematics education in technical universities represents a particularly critical domain where effective technology integration can enhance pedagogical approaches and learning outcomes for engineering students [20]. Since the initial exploration of information and communication technology (ICT) competencies for mathematics educators a decade ago [9], the technological landscape has evolved dramatically, necessitating a reassessment of the knowledge, skills, and attitudes required by contemporary mathematics educators.

The rapid pace of technological innovation in education has been further accelerated by global educational disruptions, most notably the COVID-19 pandemic, which has served as a catalyst for widespread adoption of digital teaching methodologies [13]. Mathematics educators in technical universities now operate in an environment characterized by cloud-based mathematics tools, learning analytics platforms, artificial intelligence-enhanced teaching systems, and immersive technologies – all of which present both opportunities and challenges for effective pedagogical practice.

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This paper aims to revisit and modernize the conceptual framework for digital competence development among mathematics educators in technical universities. It builds upon previous research [9] while incorporating contemporary digital competence frameworks, emerging technologies, and evolving pedagogical approaches. The objectives of this paper are to:

- Update the theoretical framework of digital competencies for mathematics educators based on current international standards and research
- Analyze the contemporary digital tools and platforms specifically relevant to mathematics education in technical universities
- Propose a multilevel competency development model that accounts for varied starting points and developmental trajectories
- Present a revised curriculum for professional development that addresses current needs and emerging trends
- Offer recommendations for support of mathematics educators' digital upskilling

2. Evolution of digital competence frameworks

2.1. From ICT literacy to digital competence

The conceptualization of educators' technology-related capabilities has evolved significantly over the past decade. Early frameworks focused primarily on technological literacy and basic operational skills with digital tools [23]. These frameworks emphasized the mastery of specific software applications and hardware operations, with limited attention to pedagogical integration and critical engagement with technology. As Caena and Redecker [2] observe, there has been a marked shift from viewing technology as a separate domain of teacher knowledge to understanding it as an integrated aspect of pedagogical practice.

The term "ICT competence" that dominated earlier discourse has progressively given way to the more comprehensive concept of "digital competence", reflecting a broader understanding of the knowledge, skills, attitudes, and values required for effective participation in digital environments [5]. This terminological shift signifies a conceptual expansion beyond technical proficiency to encompass critical evaluation, ethical considerations, and creative application of digital technologies in educational contexts.

2.2. Contemporary digital competence frameworks

Several influential frameworks have emerged in recent years to conceptualize and standardize digital competence for educators. These frameworks provide structured approaches to understanding, developing, and assessing the multifaceted capabilities required for effective teaching in digital environments.

The DigCompEdu framework, developed by the European Commission's Joint Research Centre, represents one of the most comprehensive and widely adopted approaches to conceptualizing educators' digital competence [5]. Unlike earlier frameworks that focused predominantly on technical skills, DigCompEdu addresses the specific needs of educators across educational levels and contexts.

The framework identifies six key areas of digital competence:

1. Professional engagement – using digital technologies for communication, collaboration, reflection, and professional development
2. Digital resources – selecting, creating, modifying, and managing digital resources for teaching
3. Teaching and learning – designing, planning, and implementing digital technologies in teaching and learning

4. Assessment – using digital technologies and strategies to enhance assessment
5. Empowering learners – using digital technologies to enhance inclusion, personalization, and active engagement
6. Facilitating learners' digital competence – enabling learners to creatively and responsibly use digital technologies

Each of these areas encompasses specific competencies, with progression across six proficiency levels from A1 (Newcomer) to C2 (Pioneer). This nuanced approach to proficiency development is particularly relevant for mathematics educators in technical universities, who may exhibit varying levels of competence across different domains.

The UNESCO ICT Competency Framework for Teachers (ICT-CFT) has undergone several revisions since its initial release, with the latest version reflecting contemporary technological developments and educational priorities [27]. The framework articulates the competencies required for teachers to effectively integrate ICT into their professional practice across three stages of development (Knowledge Acquisition, Knowledge Deepening, and Knowledge Creation) and six aspects of a teacher's work:

1. Understanding ICT in education
2. Curriculum and assessment
3. Pedagogy
4. Application of digital skills
5. Organization and administration
6. Teacher professional learning

The framework emphasizes the progression from basic technology literacy through knowledge deepening to knowledge creation, providing a developmental trajectory that aligns with increasing levels of educational innovation and transformation.

The International Society for Technology in Education (ISTE) Standards for Educators offer another influential framework that has evolved to reflect changing educational technologies and practices [7]. The current standards emphasize seven roles for educators in technology-enhanced environments:

1. Learner – continuously improving practice through learning with and about technology
2. Leader – supporting student empowerment and success through technology
3. Citizen – inspiring responsible participation in the digital world
4. Collaborator – collaborating with colleagues and students using digital tools
5. Designer – designing authentic learning activities enhanced by technology
6. Facilitator – facilitating learning with technology
7. Analyst – using data to improve teaching and support students

These standards are particularly relevant for mathematics educators in technical universities, as they emphasize not only technical proficiency but also leadership in technology integration and the design of authentic, discipline-specific learning experiences.

2.3. Specialized frameworks for mathematics education

While general digital competence frameworks provide valuable guidance, mathematics education presents unique challenges and opportunities for technology integration. Several researchers have proposed specialized frameworks that address the specific requirements of teaching mathematics with digital technologies.

Mishra and Koehler [14] introduced the Technological Pedagogical Content Knowledge (TPACK) framework, which has been widely applied to mathematics education.

The framework emphasizes the complex interplay between content knowledge (mathematics), pedagogical knowledge, and technological knowledge. In mathematics education, TPACK manifests as the ability to select and implement appropriate digital tools to enhance mathematical understanding and problem-solving [28].

Building on TPACK, Thurm et al. [25] identified key competencies specific to mathematics educators, including:

- Selection and evaluation of mathematics-specific digital tools (e.g., dynamic geometry software, computer algebra systems)
- Design of technology-enhanced mathematical tasks that promote conceptual understanding
- Orchestration of classroom activities that integrate digital tools with mathematical discourse
- Assessment of mathematical learning in technology-rich environments

These specialized frameworks highlight the importance of content-specific digital competencies for mathematics educators in technical universities, where the disciplinary demands of engineering mathematics require particular attention to modeling, visualization, and computational tools.

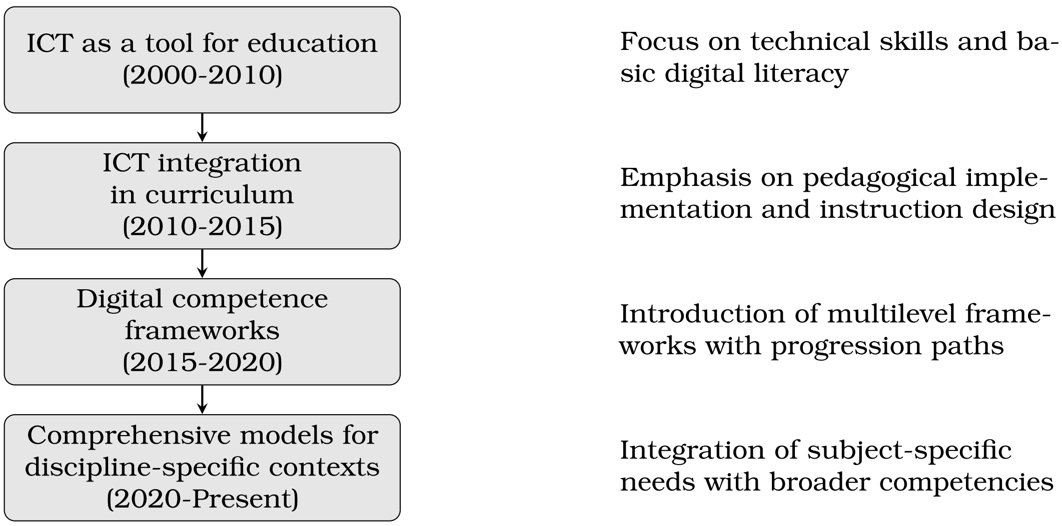

Figure 1: Evolution of digital competence frameworks over time, showing the progression from tool-focused approaches to comprehensive, discipline-specific models.

2.4. Synthesis of contemporary frameworks

The evolution of these frameworks reflects a progressive sophistication in understanding how educators engage with digital technologies (figure 1). Contemporary frameworks have shifted from technocentric approaches that focused primarily on tools and operational skills to more holistic models that emphasize pedagogical integration, critical evaluation, and creative application of technology for educational purposes.

Several common themes emerge across contemporary frameworks that are particularly relevant for mathematics educators in technical universities:

1. The integration of subject-specific considerations with general digital competencies
2. Recognition of developmental progressions in digital competence acquisition
3. Emphasis on collaboration, co-creation, and professional learning networks

4. Attention to ethical considerations, including data privacy and digital well-being
5. Focus on empowering learners through technology rather than simply delivering content

These themes inform our updated model of digital competence for mathematics educators in technical universities, presented in section 4. First, however, we examine the evolving technological landscape that shapes the context for digital competence development.

3. Contemporary digital tools and platforms for mathematics education

The technological landscape for mathematics education has evolved dramatically in the past decade, with new tools and platforms emerging that offer enhanced capabilities for visualization, computation, collaboration, and assessment. Understanding this changing technological context is essential for conceptualizing the digital competencies required by mathematics educators in technical universities.

3.1. Cloud-based mathematics tools and services

Cloud computing has transformed the accessibility and functionality of mathematics software, enabling anywhere, anytime access to powerful computational tools through web browsers and mobile devices. Key developments in cloud-based mathematics tools are presented in table 1.

Systems such as SageMathCloud (now CoCalc), Wolfram Cloud, and MATLAB Online provide web-based access to comprehensive mathematical computation environments [22]. These platforms enable educators and students to perform complex calculations, visualize mathematical objects, and develop computational models without requiring local installation of specialized software. For mathematics educators in technical universities, competence in leveraging these cloud-based systems has become increasingly important for supporting engineering applications and computational problem-solving.

Platforms such as Desmos, GeoGebra, and Maple Learn offer interactive, visual approaches to mathematical exploration and problem-solving [11]. These platforms combine computational capabilities with intuitive interfaces that allow for dynamic manipulation of mathematical objects. Mathematics educators in technical universities increasingly use these platforms to illustrate complex concepts, engage students in exploratory learning, and connect abstract mathematical ideas to engineering applications.

Table 1
Comparison of cloud-based mathematics tools and their applications in technical education.

Category	Examples	Applications in technical education
Computer algebra systems	SageMathCloud/CoCalc, Wolfram Cloud, MATLAB Online	Engineering calculations, symbolic computation, mathematical modeling, scientific computation
Interactive platforms	Desmos, GeoGebra, Maple Learn	Visualization of complex functions, geometric modeling, parameter exploration, engineering design
Mathematics LMS	WebAssign, MyMathLab, WeBWorK	Personalized practice, automated assessment, mastery-based progression, skills tracking
Collaborative environments	Overleaf, Jupyter Notebooks, Google Colab	Document preparation, collaborative problem-solving, research projects, programming assignments

Specialized learning management systems such as WebAssign, MyMathLab, and WeBWorK provide integrated environments for mathematics instruction, practice, and assessment [21]. These platforms offer automated grading of mathematical exercises, adaptive learning paths, and detailed analytics on student performance. Competence in selecting, configuring, and effectively utilizing these platforms has become a key requirement for mathematics educators in technical universities.

3.2. AI-enhanced mathematics education tools

Artificial intelligence has emerged as a transformative force in mathematics education, with AI-enhanced tools offering new capabilities for personalization, feedback, and problem-solving support.

AI-based tutoring systems such as ALEKS, Mathia, and Carnegie Learning provide personalized learning paths based on continuous assessment of student knowledge and skills [24]. These systems use adaptive algorithms to identify knowledge gaps, recommend appropriate learning activities, and provide targeted feedback.

Tools such as Photomath, Microsoft Math Solver, and Wolfram|Alpha use artificial intelligence to analyze mathematical problems, generate step-by-step solutions, and provide explanations [19]. While these tools present challenges related to academic integrity, they also offer opportunities for just-in-time support and self-directed learning.

Large language models like ChatGPT, MathGPT, and Bard have demonstrated capabilities in explaining mathematical concepts, generating practice problems, and providing alternate explanations [26]. These tools can serve as virtual teaching assistants, helping to provide individualized support to students and freeing educators to focus on higher-level instructional activities. Developing competence in effective prompting, critical evaluation of AI-generated content, and ethical use of generative AI has emerged as an important area for mathematics educators.

3.3. Immersive Technologies for Mathematical Visualization

Immersive technologies such as augmented reality (AR), virtual reality (VR), and mixed reality (MR) offer new possibilities for visualizing and interacting with mathematical objects and concepts.

Applications such as GeoGebra AR, Calcflow, and Mathland enable three-dimensional visualization and manipulation of mathematical objects in immersive environments [4]. These applications are particularly valuable for teaching concepts in multivariable calculus, linear algebra, and differential equations that involve three-dimensional structures. Mathematics educators in technical universities increasingly need competence in selecting appropriate immersive applications and designing learning activities that leverage their unique affordances.

Platforms such as PhET, Mathematica Demonstrations, and SimCalc provide interactive simulations that connect mathematical concepts to real-world applications [6]. These simulations allow students to explore mathematical models of physical phenomena, engineering systems, and scientific processes. Competence in integrating these simulations into mathematics instruction has become increasingly important for educators in technical universities.

3.4. Learning analytics and data-driven instruction

The proliferation of digital learning environments has generated unprecedented amounts of data on student engagement, performance, and learning patterns. Learning analytics platforms provide tools for collecting, analyzing, and interpreting this data to inform instructional decisions.

Platforms such as Möbius Analytics, Khan Academy Analytics, and WeBWorK Analytics provide detailed insights into student performance on mathematical tasks, identifying common misconceptions and learning patterns [12]. These platforms enable mathematics educators to identify struggling students, adapt instruction to address specific needs, and evaluate the effectiveness of learning activities. Competence in interpreting and acting on analytics data has become an essential skill for mathematics educators in technical universities.

Early warning systems such as Course Signals, E2Coach, and Blackboard Predict use predictive analytics to identify students at risk of academic difficulty [1]. These systems analyze patterns of engagement, performance, and behavior to predict student outcomes and trigger interventions. Mathematics educators in technical universities increasingly need competence in responding to early warning signals and designing appropriate interventions for struggling students.

Mapping of digital tools for mathematics education based on technological sophistication (x-axis, 1-10 scale), pedagogical integration (y-axis, 1-10 scale), and innovation potential (color gradient) is shown in figure 2. Technological sophistication represents the complexity and advancement of the technology, while pedagogical integration indicates how effectively the tool can be incorporated into teaching practices. Key examples are labeled directly on the plot. Note the trend that immersive technologies and AI tools generally show higher innovation potential (yellow) while offering varied levels of pedagogical integration.

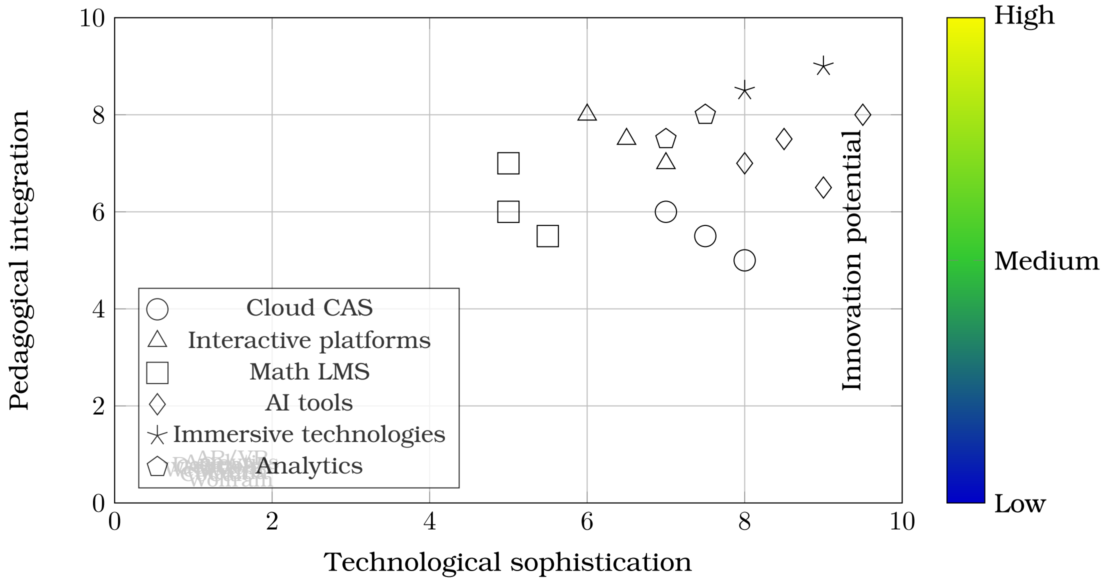

Figure 2: Mapping of digital tools for mathematics education based on technological sophistication, pedagogical integration, and innovation potential.

3.5. Blockchain and digital credentials

Emerging blockchain technologies offer new possibilities for documenting and verifying mathematical competencies and credentials.

Systems such as Open Badges, Credly, and Accredible enable the creation and verification of digital badges that represent specific mathematical skills and competencies [16]. These badges can be earned through various learning pathways and accumulated to demonstrate mastery of complex mathematical domains.

Blockchain platforms such as Blockcerts, ODEM, and Parchment enable secure, tamper-proof verification of academic credentials and achievements [17]. These systems provide a decentralized approach to credential verification that reduces fraud and enhances trust in academic qualifications. Mathematics educators in technical universities must develop understanding of these systems and their implications for certifying mathematical competencies.

3.6. Gamification and game-based learning

Gamification strategies and game-based learning approaches offer engaging ways to develop mathematical skills and motivate learners.

Platforms such as Prodigy, Mathigon, and Brilliant incorporate game mechanics such as points, levels, badges, and leaderboards to motivate engagement with mathematical content [15]. These platforms can increase student motivation and time-on-task, particularly for foundational skills development.

Games such as DragonBox, Euclidea, and Satisfactory apply game-based approaches to mathematical problem-solving and concept development [10]. These games embed mathematical thinking within engaging contexts that motivate sustained engagement. Competence in identifying and leveraging appropriate mathematical games has become an important aspect of the digital toolkit for mathematics educators in technical universities.

The evolving technological landscape presents both opportunities and challenges for mathematics educators in technical universities. Effective integration of these diverse tools and platforms requires not only technical proficiency but also pedagogical creativity, critical evaluation skills, and an understanding of how different technologies can support specific learning objectives. In the next section, we present an updated model of digital competence that addresses these requirements.

4. Updated model of digital competence for mathematics educators

Building on contemporary digital competence frameworks and the evolving technological landscape, we present an updated model of digital competence specifically tailored to mathematics educators in technical universities. This model integrates the general digital competencies required of all educators with the specialized competencies needed for effective mathematics instruction in technical contexts.

4.1. Core components of digital competence

Our model identifies six core components of digital competence for mathematics educators in technical universities:

4.1.1. Professional digital engagement

This component encompasses the use of digital technologies for professional communication, collaboration, and continuous development. For mathematics educators in technical universities, this includes:

- Participation in online professional communities focused on mathematics education, such as the Math Forum, Mathematics Education Network, or discipline-specific communities in engineering mathematics
- Use of digital tools for collaboration with colleagues, including co-creation of learning resources and collaborative research projects
- Engagement with digital sources of professional knowledge, such as online journals, webinars, and MOOCs related to mathematics education and technology integration
- Reflective use of digital technologies to analyze and improve teaching practice, including video analysis, learning analytics, and student feedback systems

4.1.2. Mathematical digital resources

This component focuses on the selection, creation, modification, and management of digital resources specifically designed for mathematics instruction. Key competencies include:

- Evaluation and selection of appropriate digital resources for specific mathematical topics and learning objectives, considering mathematical accuracy, pedagogical approach, and technical accessibility
- Creation and adaptation of digital mathematics resources using appropriate authoring tools, such as interactive worksheets, mathematical visualizations, and computational models
- Curation and organization of digital resources for mathematics instruction, including the development of coherent resource collections aligned with curriculum goals
- Sharing and licensing of mathematical resources, including understanding of copyright issues and open educational resource (OER) approaches

4.1.3. Technology-enhanced mathematics pedagogy

This component addresses the design, planning, and implementation of digital technologies in mathematics teaching and learning. For mathematics educators in technical universities, this includes:

- Design of technology-enhanced learning activities that leverage digital tools to promote mathematical understanding, problem-solving, and application to engineering contexts
- Orchestration of digital learning environments, including management of individual, collaborative, and whole-class activities with digital tools
- Implementation of student-centered approaches that use technology to support active learning, inquiry, and problem-based learning in mathematics
- Integration of authentic engineering applications that use digital tools to connect mathematical concepts to real-world engineering problems

4.1.4. Digital assessment in mathematics

This component focuses on the use of digital technologies for assessment of mathematical learning. Key competencies include:

- Design of digital assessment tasks that effectively evaluate mathematical understanding, including conceptual knowledge, procedural fluency, and problem-solving skills
- Implementation of formative assessment strategies using digital tools, such as classroom response systems, digital quizzes, and interactive problem sets
- Analysis and interpretation of assessment data from digital platforms to inform instructional decisions and provide targeted feedback
- Development of authentic assessment approaches that use digital tools to evaluate application of mathematics in engineering contexts

4.1.5. Empowering mathematics learners

This component addresses the use of digital technologies to enhance accessibility, personalization, and active engagement in mathematics learning. For educators in technical universities, this includes:

- Implementation of differentiated learning approaches using digital tools to address diverse needs and abilities in mathematics classrooms
- Design of personalized learning pathways that use adaptive technologies to provide appropriate challenges and support for individual learners
- Promotion of student agency and self-regulation through digital tools that support goal-setting, progress monitoring, and reflection on mathematical learning
- Cultivation of collaborative mathematical practices through digital platforms that enable peer interaction, group problem-solving, and knowledge co-construction

4.1.6. Facilitating mathematical digital competence

This component focuses on enabling learners to develop their own digital competence in mathematical contexts. Key competencies include:

- Promotion of responsible use of digital tools for mathematical learning, including academic integrity, critical evaluation of digital resources, and appropriate attribution of sources
- Development of students' computational thinking through programming, algorithm design, and data analysis activities that connect to mathematical concepts
- Cultivation of data literacy through activities that engage students in collecting, analyzing, and interpreting data using digital tools
- Support for creative problem-solving with digital tools, including the use of modeling software, simulation environments, and visualization tools

4.2. Multilevel competency development model

Recognizing that mathematics educators enter the profession with diverse backgrounds and experiences with digital technologies, our model proposes a multilevel approach to competency development. Building on the progression levels in the DigCompEdu framework, we adapt these to the specific context of mathematics education in technical universities (figure 3).

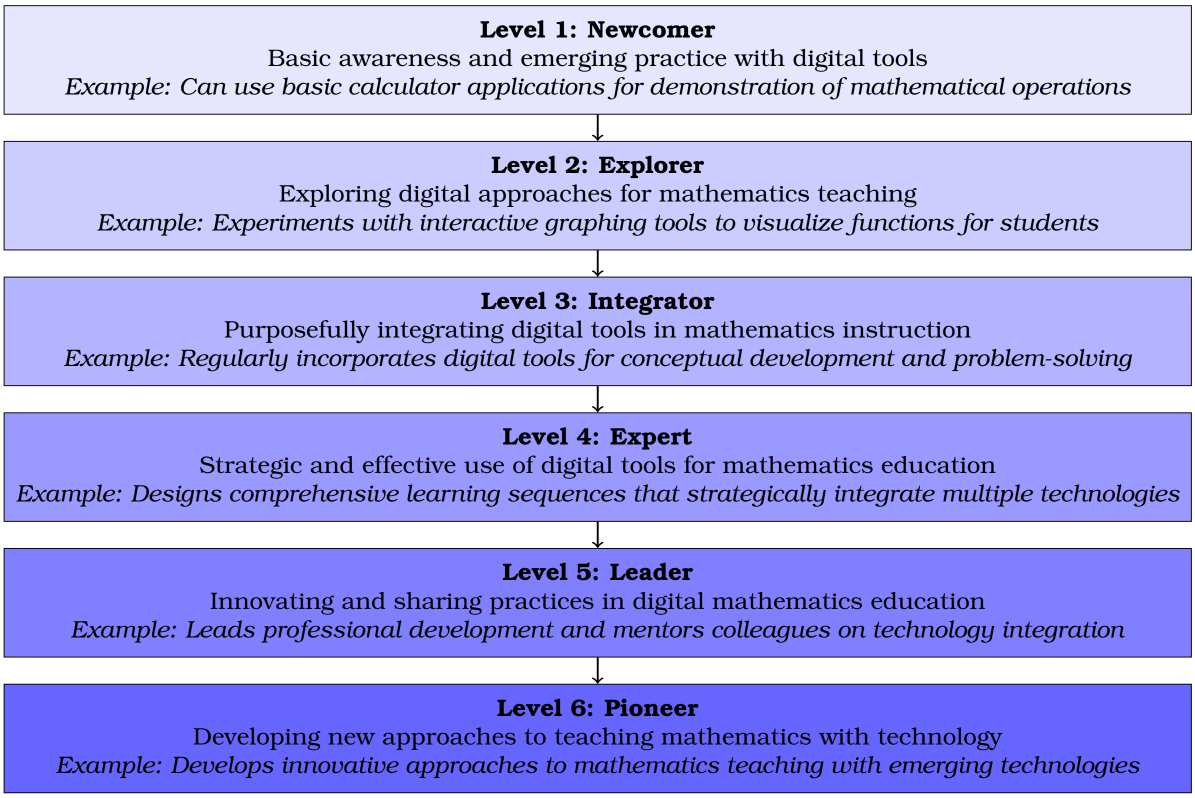

Figure 3: Multilevel model of digital competence development for mathematics educators in technical universities, showing progression from basic awareness to innovative practice.

Level 1: Newcomer

At this level, mathematics educators demonstrate basic awareness and emerging practice with digital tools. They may use simple digital resources such as presentation software, calculator applications, and basic learning management system functions. Their approach to technology integration tends to be teacher-centered, with digital tools primarily used to support traditional teaching methods rather than transform pedagogical practice. Educators at this level benefit from structured guidance, hands-on workshops, and access to ready-to-use resources that provide entry points to technology integration.

Level 2: Explorer

Educators at the Explorer level have begun to experiment with a wider range of digital tools for mathematics teaching. They explore interactive resources such as dynamic geometry software, online graphing tools, and digital assessment platforms. While their use of technology remains somewhat tentative and episodic, they demonstrate increasing curiosity about the potential of digital tools to enhance student learning. Explorers benefit from peer collaboration, targeted professional development, and opportunities to observe effective technology integration in mathematics classrooms.

Level 3: Integrator

Integrators have developed confidence and competence in incorporating digital tools into their regular teaching practice. They purposefully select and implement technologies that align with specific learning objectives and pedagogical strategies. Their approach balances teacher-directed and student-centered activities, with increasing attention to how technology can support active learning and student engagement. Integrators benefit from professional learning communities, subject-specific technology training, and support for curriculum redesign that effectively incorporates digital tools.

Level 4: Expert

Experts demonstrate sophisticated understanding and strategic use of digital technologies for mathematics education. They select and combine multiple tools to create coherent learning experiences that address diverse student needs and promote deeper mathematical understanding. Their approach is predominantly student-centered, with technology used to support inquiry, problem-solving, and authentic application. Experts continuously refine their practice based on critical reflection and evidence of student learning. They benefit from advanced professional development, participation in innovation networks, and opportunities to conduct action research on technology integration.

Level 5: Leader

Leaders not only excel in their own technology integration practice but also guide and inspire colleagues to enhance their digital competence. They lead professional development initiatives, mentor other educators, and contribute to the development of institutional policies and practices related to technology integration. Leaders advocate for effective use of digital technologies in mathematics education and promote evidence-based approaches within their institutions and professional communities. They benefit from leadership development opportunities, participation in regional or national initiatives, and connections with researchers in mathematics education and educational technology.

Level 6: Pioneer

Pioneers represent the cutting edge of innovation in teaching mathematics with technology. They experiment with emerging technologies, develop new pedagogical approaches, and challenge conventional practices in mathematics education. Pioneers conduct and publish research on technology integration, contribute to the development of new digital tools and resources, and influence policy and practice at institutional, national, or international levels. They benefit from research collaborations, partnerships with technology developers, and opportunities to showcase and disseminate innovative practices.

This multilevel model provides a developmental pathway for mathematics educators to progressively enhance their digital competence. It recognizes that educators may be at different levels across different competency components, reflecting their unique backgrounds, interests, and professional contexts. The model can be used to guide individual professional development planning, institutional capacity building, and the design of targeted support structures for educators at different stages of digital competence development.

4.3. Competency matrix for mathematics education in technical universities

To provide a more detailed framework for assessing and developing digital competence, we propose a competency matrix that integrates the six core components with the six developmental levels. Table 2 presents an excerpt from this matrix, focusing on the component of technology-enhanced mathematics pedagogy.

Table 2
Competency matrix for technology-enhanced mathematics pedagogy.

Level	Competency description	Example in technical mathematics context
Newcomer	Uses basic digital tools to present mathematical content, primarily in a teacher-centered approach	Creates PowerPoint presentations to display mathematical formulas and worked examples for engineering applications
Explorer	Experiments with interactive technologies to demonstrate mathematical concepts and engage students	Uses GeoGebra occasionally to visualize geometric transformations relevant to engineering design
Integrator	Regularly incorporates a range of digital tools in mathematics instruction, balancing teacher-directed and student-centered approaches	Designs learning sequences that integrate computational tools for solving differential equations in circuit analysis
Expert	Designs comprehensive learning environments that strategically integrate multiple technologies to support diverse learning needs and deepen mathematical understanding	Creates blended learning modules that combine simulation tools, computational software, and collaborative problem-solving for engineering mathematics
Leader	Guides colleagues in effective technology integration, develops exemplary learning resources, and contributes to departmental initiatives for technology-enhanced mathematics education	Leads a faculty learning community on integrating computer algebra systems across the engineering mathematics curriculum
Pioneer	Develops innovative approaches to teaching mathematics with technology, conducts research on effectiveness, and influences broader educational practice	Designs and evaluates immersive virtual reality environments for teaching multivariable calculus concepts in engineering contexts

Similar matrices can be developed for each of the six core components, providing a comprehensive framework for assessing current competence levels and planning targeted professional development. The complete matrix serves as a reference for educational leaders, professional development providers, and mathematics educators themselves in charting pathways for digital competence development.

5. Revised curriculum for professional development

Building on the updated model of digital competence, we propose a revised curriculum for professional development that addresses the contemporary needs of mathematics educators in technical universities. This curriculum is designed to support the development of digital competence across multiple levels, with differentiated pathways to accommodate diverse starting points and professional goals.

5.1. Curriculum structure and design principles

The proposed curriculum follows a modular structure, allowing for flexible implementation across various professional development formats, including workshops, courses, communities of practice, and self-directed learning. The curriculum is guided by several key design principles:

- **Practice-based learning** – emphasis on authentic tasks and problems of practice that connect digital competence development to the daily work of mathematics educators
- **Differentiated pathways** – multiple entry points and progression routes to accommodate diverse backgrounds, experiences, and professional needs
- **Just-in-time learning** – opportunities to access targeted support and resources at the point of need, rather than solely through predefined sequences
- **Collaborative inquiry** – structured opportunities for educators to learn with and from colleagues through collaborative exploration and reflection
- **Sustained engagement** – extended learning experiences that allow for implementation, reflection, and refinement of practice over time
- **Integration of theory and practice** – connections between research-based principles and practical applications in mathematics education contexts

5.2. Core curriculum modules

The revised curriculum comprises eight core modules, each addressing key aspects of digital competence for mathematics educators in technical universities (figure 4). These modules can be implemented individually or in combination, depending on institutional needs and available resources.

Module 1: Digital landscape for mathematics education

This foundational module introduces the contemporary digital landscape for mathematics education, with emphasis on tools and platforms particularly relevant to technical and engineering contexts. Topics include:

- Overview of digital tools for mathematics education, including computer algebra systems, interactive visualization platforms, and computational environments
- Analysis of affordances and constraints of different digital tools for specific mathematical topics and learning objectives
- Evaluation criteria for selecting appropriate digital tools for different instructional purposes
- Emerging technologies and their potential implications for mathematics education in technical contexts

This module supports educators in developing a comprehensive understanding of available digital tools and making informed decisions about which technologies best support specific learning goals.

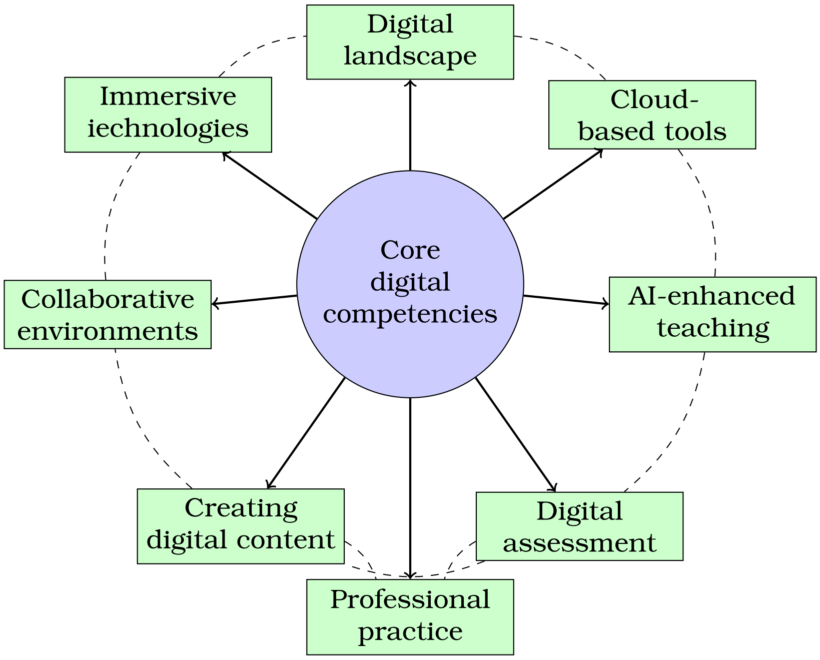

Figure 4: Modular curriculum structure showing connections between core digital competencies and specialized modules for mathematics educators.

Module 2: Cloud-based mathematics tools and platforms

This module focuses on the effective use of cloud-based tools for mathematics education, with particular emphasis on tools relevant to technical and engineering contexts. Topics include:

- Implementation of cloud-based computer algebra systems such as CoCalc, Wolfram Cloud, and MATLAB Online
- Integration of interactive mathematics platforms such as Desmos, GeoGebra, and Maple Learn
- Use of collaborative environments such as Jupyter Notebooks, Overleaf, and Google Colab
- Management of cloud-based resources, including organization, sharing, and version control

This module develops educators' technical skills with cloud-based mathematics tools while emphasizing pedagogical approaches that leverage these tools for enhanced student learning.

Module 3: AI-enhanced mathematics teaching and learning

This module explores the integration of artificial intelligence tools in mathematics education, addressing both opportunities and challenges. Topics include:

- Implementation of intelligent tutoring systems and adaptive learning platforms for mathematics instruction
- Strategies for effectively incorporating AI-powered problem-solving tools into mathematics courses
- Approaches to leveraging generative AI for content creation, explanation, and assessment in mathematics education
- Ethical considerations and academic integrity issues related to AI tools in mathematics education

This module prepares educators to navigate the rapidly evolving landscape of AI-enhanced education, making strategic decisions about how to leverage these tools while maintaining educational quality and integrity.

Module 4: Digital assessment and feedback in mathematics

This module focuses on effective approaches to assessment and feedback using digital tools, with emphasis on evaluating mathematical understanding and problem-solving in technical contexts. Topics include:

- Design of effective digital assessment tasks for mathematics, including automated and semi-automated approaches
- Implementation of formative assessment strategies using digital tools and platforms
- Use of learning analytics to monitor student progress and identify areas for intervention
- Approaches to providing meaningful feedback on mathematical work in digital environments

This module supports educators in developing comprehensive assessment approaches that leverage digital tools to provide timely, targeted feedback and enhance student learning.

Module 5: Creating digital mathematics content

This module develops educators' skills in creating digital content for mathematics education, with emphasis on resources that support technical and engineering applications. Topics include:

- Development of interactive mathematics materials using authoring tools and platforms
- Creation of instructional videos and multimedia resources for mathematics education
- Design of computational notebooks and interactive textbooks for mathematics courses
- Approaches to ensuring accessibility and inclusivity in digital mathematics content

This module enhances educators' capacity to create high-quality digital resources that address specific learning needs and contexts in technical mathematics education.

Module 6: Collaborative and Interactive Learning Environments

This module focuses on designing and implementing collaborative learning experiences using digital tools, with emphasis on developing mathematical collaboration and communication skills. Topics include:

- Design of collaborative problem-solving activities using digital platforms
- Implementation of team-based learning approaches in digital and hybrid environments
- Facilitation of mathematical discourse and argumentation in online spaces
- Strategies for monitoring and assessing collaborative work in digital environments

This module prepares educators to create learning environments that develop not only mathematical knowledge and skills but also the collaborative capabilities essential for professional practice in technical fields.

Module 7: Immersive Technologies for Mathematical Visualization

This module explores the use of immersive technologies such as augmented and virtual reality for enhancing mathematical visualization and understanding. Topics include:

- Implementation of AR/VR applications for visualizing three-dimensional mathematical objects and relationships
- Design of learning activities that leverage immersive technologies for conceptual understanding
- Integration of interactive simulations and virtual laboratories in mathematics education
- Approaches to evaluating student learning in immersive environments

This module develops educators' capacity to use emerging immersive technologies to address persistent challenges in visualizing abstract mathematical concepts, particularly in multidimensional contexts relevant to engineering applications.

Module 8: Digital professional practice and leadership

This module focuses on digital aspects of professional practice and leadership in mathematics education, supporting educators in developing their professional digital identity and contributing to educational innovation. Topics include:

- Participation in online professional communities and networks for mathematics educators
- Use of digital tools for reflective practice and professional learning
- Approaches to mentoring colleagues and leading professional development in technology integration
- Strategies for contributing to scholarship and innovation in digital mathematics education

This module supports educators in developing not only their individual digital competence but also their capacity to contribute to broader educational improvement through collaboration, leadership, and scholarship.

5.3. Implementation approaches

The modular curriculum can be implemented through various approaches, depending on institutional contexts, available resources, and educator needs. Potential implementation strategies include:

1. *Comprehensive professional development program* – a structured program that systematically addresses all modules through a combination of workshops, online courses, and applied projects. This approach provides a comprehensive pathway for digital competence development, with opportunities for educators to progress through multiple levels over an extended period (e.g., 1-2 years). The program can include cohort-based learning, mentoring relationships, and opportunities for educators to document and showcase their developing practice.
2. *Topic-focused workshops and communities of practice* – targeted workshops that address specific modules or topics, complemented by ongoing communities of practice that support implementation and reflection. This approach allows for focused attention to particular aspects of digital competence based on institutional priorities or educator interests. Communities of practice provide sustained support for implementing new approaches and solving problems of practice that emerge during implementation.

3. *Self-directed learning with coaching support* – resource collections and learning pathways that enable educators to engage in self-directed learning based on their individual needs and interests, supported by coaching from experienced technology integrators. This approach provides maximum flexibility for educators with diverse backgrounds and constraints, while still ensuring access to guidance and feedback from knowledgeable mentors.
4. *Collaborative action research* – structured inquiry projects in which educators collaboratively investigate questions related to technology integration in mathematics education, designing, implementing, and evaluating innovations in their teaching practice. This approach emphasizes the development of evidence-based practice and contributes to the scholarship of teaching and learning in mathematics education.

5.4. Assessment and recognition

The curriculum includes multiple approaches to assessment and recognition of digital competence development:

1. *Competency-based assessment* – evaluation of specific digital competencies through performance tasks, portfolios, and other authentic assessments. This approach focuses on demonstrating proficiency in particular competencies rather than simply completing activities or accruing time in professional development.
2. *Digital badges and micro-credentials* – recognition of specific competencies and achievements through digital badges and micro-credentials that can be shared across professional platforms. This approach provides visible recognition of incremental progress and allows educators to build a comprehensive digital portfolio of their developing competence.
3. *Professional certification* – formal certification programs that recognize comprehensive digital competence for mathematics education, potentially aligned with institutional, national, or international standards. This approach provides external validation of educator competence and may be linked to career advancement opportunities.
4. *Blockchain-based credentials* – secure, verifiable records of professional learning and competence development using blockchain technology. This approach ensures the integrity and portability of credentials while potentially enabling more granular and multidimensional representations of educator competence.

The revised curriculum provides a comprehensive framework for supporting the digital competence development of mathematics educators in technical universities. By addressing both fundamental and emerging aspects of digital practice, the curriculum prepares educators to effectively leverage technology for enhanced teaching and learning in contemporary educational contexts.

6. Recommendations

Effective development of digital competence among mathematics educators requires supportive institutional environments and enabling policy frameworks. Drawing on our analysis of contemporary digital competence frameworks, technological trends, and professional development approaches, we offer recommendations for institutions and policymakers seeking to enhance digital competence in mathematics education.

6.1. Recommendations for technical universities

1. *Strategic planning and leadership*

- Develop a comprehensive digital strategy for mathematics education that aligns with broader institutional goals and disciplinary requirements of technical and engineering programs
- Establish clear expectations for digital competence among mathematics faculty, while recognizing diverse starting points and providing differentiated support
- Identify and empower digital champions within mathematics departments who can lead by example and support colleagues in technology integration
- Promote cross-disciplinary collaboration between mathematics educators and colleagues in computer science, engineering, and educational technology

2. *Infrastructure and resources*

    - Ensure adequate technological infrastructure to support digital teaching and learning, including reliable network connectivity, appropriate hardware, and necessary software licenses
    - Provide accessible and user-friendly learning management systems with specific functionality for mathematics education, such as equation editing, computational capabilities, and interactive visualization
    - Establish digital resource repositories for mathematics education that enable faculty to discover, share, and adapt high-quality learning materials
    - Invest in specialized technologies for mathematics education, including computer algebra systems, interactive visualization tools, and immersive technologies

3. *Professional development and support*

    - Implement the revised curriculum for digital competence development, adapted to institutional contexts and faculty needs
    - Provide ongoing technical and pedagogical support for mathematics educators, including just-in-time assistance, peer mentoring, and specialized consultations
    - Establish communities of practice focused on technology integration in mathematics education, facilitating knowledge sharing and collaborative problem-solving
    - Recognize and reward innovations in digital teaching, including consideration of technology integration in promotion and tenure processes

4. *Assessment and quality assurance*

    - Develop institutional frameworks for assessing digital competence among mathematics educators, aligned with the multilevel model presented in this paper
    - Implement regular evaluations of digital learning environments and resources for mathematics education, focusing on accessibility, usability, and pedagogical effectiveness
    - Collect and analyze data on student engagement and outcomes in technology-enhanced mathematics courses to inform continuous improvement
    - Participate in national and international benchmarking initiatives related to digital teaching and learning in mathematics

6.2. Recommendations for national educational policies

1. *Policy frameworks and standards*
    - Develop national frameworks for digital competence in mathematics education, providing clear guidance while allowing for contextual adaptation
    - Establish standards for digital resources and platforms used in mathematics education, addressing issues of quality, accessibility, and interoperability
    - Integrate digital competence into professional standards for mathematics educators, recognizing its essential role in contemporary teaching practice
    - Align teacher education and certification requirements with digital competence frameworks to ensure adequate preparation of new mathematics educators
2. *Funding and resources*
    - Provide targeted funding for digital infrastructure and resources that support mathematics education in technical universities
    - Invest in professional development programs that enhance digital competence among mathematics educators, with particular attention to innovative approaches and emerging technologies
    - Support research and development initiatives focused on innovative digital tools and approaches for mathematics education
    - Establish national repositories of open educational resources for mathematics, facilitating access to high-quality digital materials
3. *Research and innovation*
    - Fund research on effective approaches to developing digital competence among mathematics educators, with attention to discipline-specific needs and contexts
    - Support pilot projects and experimental initiatives that explore innovative approaches to technology integration in mathematics education
    - Facilitate knowledge exchange and dissemination of effective practices through conferences, publications, and online platforms
    - Establish partnerships between educational institutions, technology developers, and industry to address emerging needs in mathematics education
4. *International collaboration*
    - Participate in international initiatives related to digital competence in education, such as those led by UNESCO, the European Commission, and the OECD
    - Facilitate cross-national collaboration on research, development, and implementation of digital approaches to mathematics education
    - Support the participation of mathematics educators in international professional networks and communities of practice
    - Engage in benchmarking and comparative analysis to identify effective policies and practices from other educational systems

7. Emerging trends and future directions

As we look toward the future of digital competence for mathematics educators in technical universities, several emerging trends and directions merit attention. These developments are likely to shape the evolving landscape of mathematics education and influence the competencies required of educators in the coming years.

7.1. Blockchain technology for credentialing and digital identity

Blockchain technology offers promising applications for documenting and verifying educational achievements, professional credentials, and digital competencies [16]. For mathematics educators, blockchain-based systems could provide secure, tamper-proof records of professional learning experiences, competency development, and educational innovations. These systems could enable more granular and multidimensional representations of digital competence than traditional certification approaches, allowing for recognition of specialized expertise in areas such as computational mathematics, visualization techniques, or discipline-specific applications.

Furthermore, blockchain technology has potential applications in student credential verification, providing secure and transparent documentation of mathematical knowledge and skills. Mathematics educators will increasingly need to understand these systems and their implications for certifying student learning, particularly in contexts where traditional assessment approaches are being reimagined for digital environments.

7.2. Artificial intelligence and adaptive learning systems

Artificial intelligence continues to transform educational practices, with increasingly sophisticated applications for mathematics education [19]. For mathematics educators, AI presents both challenges and opportunities:

- AI-enhanced tutoring systems can provide personalized support for students, potentially freeing educators to focus on higher-level aspects of mathematics instruction
- Generative AI tools can assist in creating customized learning resources, providing alternative explanations, and generating practice problems aligned with specific learning objectives
- AI-powered analytics can provide deeper insights into student learning patterns and misconceptions, enabling more targeted interventions and instructional adjustments
- Learning engineering approaches that combine AI, cognitive science, and educational design can lead to more effective and engaging mathematics learning experiences

Mathematics educators will increasingly need competence not only in using AI tools but also in critically evaluating their capabilities and limitations, designing appropriate tasks that leverage AI while maintaining academic integrity, and guiding students in responsible use of AI for mathematical learning.

7.3. Extended reality and spatial computing

Extended reality (XR) technologies, including augmented reality (AR), virtual reality (VR), and mixed reality (MR), offer promising approaches to addressing persistent challenges in mathematics education, particularly related to visualization and spatial understanding [4]. As these technologies become more accessible and integrated into educational environments, mathematics educators will need new competencies:

- Selection and evaluation of XR applications for specific mathematical topics and learning objectives
- Design of learning activities that effectively leverage immersive environments for conceptual understanding
- Orchestration of learning experiences that integrate physical and virtual elements
- Assessment of student learning in immersive environments

The emergence of spatial computing platforms that seamlessly blend digital and physical environments will further expand possibilities for mathematics education, enabling new approaches to visualizing abstract concepts, exploring mathematical relationships, and connecting mathematical ideas to real-world contexts.

7.4. Gamification and game-based learning

Gamification strategies and game-based learning approaches continue to evolve as tools for engaging students and enhancing motivation in mathematics education [15]. Emerging trends in this area include:

- Integration of narrative elements and immersive storytelling to contextualize mathematical problems and concepts
- Development of adaptive game-based systems that dynamically adjust challenges based on student performance and engagement
- Implementation of collaborative game environments that develop both mathematical understanding and teamwork skills
- Creation of maker-centered approaches that combine game design with mathematical modeling and problem-solving

Mathematics educators will increasingly need competence in selecting and implementing appropriate game-based approaches, designing effective learning activities that leverage game mechanics, and evaluating the impact of gamification on student engagement and learning outcomes.

7.5. Personalized learning paths and competency-based education

Personalized learning approaches that adapt to individual student needs, interests, and learning patterns continue to gain prominence in mathematics education [8]. These approaches often involve competency-based frameworks that focus on demonstration of mastery rather than time-based progression. For mathematics educators, this shift necessitates new competencies:

- Design of learning pathways that allow for multiple routes to mathematical understanding
- Implementation of formative assessment approaches that provide timely feedback and guide personalized learning
- Orchestration of learning environments that support diverse activities and progression rates
- Use of learning analytics to monitor student progress and make data-informed decisions about instructional interventions

As personalized and competency-based approaches become more prevalent, mathematics educators will need to balance individualization with collaborative learning experiences that develop communication, cooperation, and collective problem-solving skills essential for professional practice in technical fields.

7.6. Collaborative and networked learning

Digital technologies continue to transform possibilities for collaboration and knowledge construction in mathematics education [3]. Emerging trends in collaborative and networked learning include:

- Development of online communities of practice that connect students with professionals in mathematics-intensive fields

- Implementation of collaborative problem-solving environments that support synchronous and asynchronous mathematical work
- Creation of participatory learning experiences that involve students in authentic research and application projects
- Emergence of globally networked classrooms that enable cross-institutional and cross-cultural mathematical collaboration

Mathematics educators will increasingly need competence in designing and facilitating effective collaborative learning experiences, managing digital environments for mathematical discourse and co-creation, and guiding students in developing productive collaborative practices.

7.7. Data science and computational thinking integration

The growing importance of data science and computational thinking in technical and engineering fields continues to influence mathematics education [18]. Emerging trends in this area include:

- Integration of computational approaches to mathematical problem-solving, including programming, simulation, and data analysis
- Development of interdisciplinary learning experiences that connect mathematics with data science, computer science, and domain applications
- Implementation of project-based approaches that engage students in authentic data analysis and modeling tasks
- Emphasis on statistical literacy and critical evaluation of data-driven claims

Mathematics educators in technical universities will increasingly need competence in computational tools and approaches, interdisciplinary collaboration, and the design of learning experiences that develop both mathematical understanding and computational thinking skills.

7.8. Implications for digital competence development

These emerging trends suggest several implications for the development of digital competence among mathematics educators in technical universities:

- The scope of required technical knowledge continues to expand, necessitating ongoing professional learning and specialized expertise development
- Pedagogical approaches must evolve to leverage new technological capabilities while maintaining focus on essential mathematical learning outcomes
- Collaborative professional learning becomes increasingly important as a strategy for navigating complex and rapidly changing technological environments
- Critical evaluation skills are essential for assessing the affordances, limitations, and ethical implications of emerging technologies
- Adaptability and openness to innovation remain core meta-competencies for effective teaching in digital environments

Professional development programs and institutional support structures must continue to evolve alongside these emerging trends, providing mathematics educators with opportunities to explore, experiment, and develop competence with new technologies and approaches.

8. Conclusion

This paper has presented an updated conceptual framework for digital competence development among mathematics educators in technical universities. Building on earlier work [9] and incorporating contemporary frameworks, technological developments, and pedagogical approaches, we have offered a comprehensive model that addresses the current and emerging needs of mathematics education in technical contexts.

The multilevel competency model provides a developmental pathway that recognizes diverse starting points and progression routes, while the core components of digital competence offer a structured framework for assessing and developing specific capabilities essential for effective mathematics teaching with technology. The revised curriculum for professional development translates these conceptual elements into practical learning experiences that can be implemented in various institutional contexts.

As mathematics education continues to evolve in response to technological innovations and changing educational demands, the development of digital competence among educators remains a critical priority. The framework presented in this paper provides a foundation for systematic and strategic approaches to this development, supporting both individual professional growth and institutional capacity building.

Future research should focus on empirical validation of the proposed framework, exploration of effective implementation strategies in diverse institutional contexts, and investigation of the impact of digital competence development on teaching practices and student outcomes. Additionally, continued attention to emerging technologies and pedagogical approaches will be essential for maintaining the relevance and effectiveness of digital competence frameworks in an era of rapid technological change.

Declaration on generative AI: During the preparation of this work, the author used Claude 3.7 Sonnet to enhance content. After using this tool, the author reviewed and edited the content as needed and took full responsibility for the publication's content.

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