A relativistic approach to teaching electrodynamics: Deriving Maxwell's equations from first principles

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Abstract

This paper presents a novel methodology for teaching classical electrodynamics based on special relativity theory. Traditional approaches often rely heavily on empirical laws, potentially obscuring the fundamental unity of electromagnetic phenomena. We demonstrate how the core principles of electrodynamics, including Maxwell's equations, can be derived from just two fundamental postulates: the principle of relativity and Coulomb's law. By consistently applying relativistic reasoning, we show how key concepts such as the Lorentz force, Biot-Savart law, and electromagnetic induction emerge naturally. This approach provides a logically coherent framework for understanding electrodynamics, emphasizes its inherently relativistic nature, and resolves apparent paradoxes that arise in non-relativistic treatments. We discuss the pedagogical advantages of this method, including its potential to foster deeper conceptual understanding and develop students' theoretical reasoning skills. The paper also addresses practical aspects of implementing this approach in higher education physics curricula and its implications for physics education research.

Keywords

electrodynamics education, special relativity, Maxwell's equations, physics pedagogy, relativistic electromagnetism, theoretical physics education, electromagnetic induction, Lorentz force, Biot-Savart law, fundamentalization of physics education

> Our experience up to date justifies us in feeling sure that in Nature is actualized the ideal of mathematical simplicity. It is my conviction that pure mathematical construction enables us to discover the concepts and the laws connecting them which give us the key to the understanding of the phenomena of Nature. ... It is essential for our point of view that we can arrive at these constructions and the laws relating them one with another by adhering to the principle of searching for the mathematically simplest concepts and their connections. In the paucity of the mathematically existent simple field-types and of the relations between them, lies the justification for the theorist's hope that he may comprehend reality in its depths.
>
> *On the Method of Theoretical Physics* [7, p. 167-168]
> Albert Einstein

1. Introduction

This paper presents a novel approach to teaching electrodynamics based on special relativity theory. The traditional methods of teaching electrodynamics in higher

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education often rely heavily on empirical laws and do not fully integrate relativistic concepts. This can lead to inconsistencies and a lack of deep understanding of the fundamental nature of electromagnetic phenomena.

The approach outlined here aims to derive the core principles of electrodynamics from just two fundamental postulates: the principle of relativity and Coulomb's law. By consistently applying relativistic reasoning, we show how the key laws and equations of electrodynamics – including the Lorentz force law, Biot-Savart law, Ampère's law, and Maxwell's equations – can be obtained as logical consequences of these basic principles.

This method offers several pedagogical advantages:

1. It provides a logically coherent framework for understanding electrodynamics, demonstrating how the various laws are interconnected rather than existing as separate empirical facts.
2. It emphasizes the inherently relativistic nature of electromagnetism, giving students a deeper appreciation for the role of special relativity in classical physics.
3. It develops students' skills in theoretical reasoning and deduction, moving beyond rote learning of equations.
4. It resolves some apparent contradictions and paradoxes that arise in non-relativistic treatments.

The paper begins by analyzing the electromagnetic field of a uniformly moving charged particle, deriving expressions for the electric and magnetic fields. This serves as a foundation for understanding more complex electromagnetic phenomena. We then examine the interaction between two moving charged particles, showing how this leads naturally to concepts like the Lorentz force and magnetic field.

Building on these results, we derive the Biot-Savart law in its relativistic form and show how it reduces to the classical form in the appropriate limit. The principle of relativity is consistently applied to ensure that our descriptions are physically sensible in all inertial reference frames.

We pay particular attention to phenomena like electromagnetic induction, demonstrating how they arise as necessary consequences of relativity and Coulomb's law. This approach provides new insight into the physical mechanisms underlying these effects.

The paper concludes by deriving Maxwell's equations in vacuum, showing how they encapsulate all the previous results in a concise set of field equations. Throughout, we emphasize the logical structure of the theory and its grounding in fundamental principles.

By presenting electrodynamics in this way, we aim to foster a deeper conceptual understanding of the subject among students and equip them with powerful tools for analyzing electromagnetic phenomena. This approach aligns with modern pedagogical principles emphasizing the importance of fundamental understanding over mere calculation, preparing students for advanced study and research in physics.

2. Overview of methods for deriving Maxwell's equations

According to the traditional definition, electrodynamics studies the interaction of charged particles and electric currents with each other and with the electromagnetic field.

Maxwell's equations establish the relationship between the distribution and motion of charged particles and the quantities that characterize the electromagnetic field, and constitute the essence of electromagnetic field theory. Therefore, deriving Maxwell's equations and revealing their physical meaning is always an important methodological problem when studying both classical and relativistic electrodynamics.

A review of the educational and methodological literature shows that the following methods are mainly used to derive Maxwell's equations:

A. Maxwell's equations are obtained by generalizing (inductive method) based on the laws of Coulomb, Biot-Savart, Ampère-Grassmann, electromagnetic induction (EMI), charge conservation, which are considered fundamental in traditional methods of teaching electrodynamics [28, 31, 46, 62]. But we have shown (see sections 5-8) that the laws of Biot-Savart, Ampère-Grassmann, electromagnetic induction are consequences of the principle of relativity and Coulomb's law.

B. In each separate section of electrodynamics (electrostatics, electromagnetic waves, etc.), those Maxwell equations that describe the corresponding range of phenomena are derived. From this, a generalizing conclusion is made about the existence of a system of Maxwell's equations that describes the entire set of electrodynamic phenomena [15, 47, 53, 57, 60, 61].

The difference between methods A and B is only that the above-mentioned "fundamental" experimental facts in method A are considered all at once and as a result of analysis and generalization, Maxwell's equations are obtained [31, 46, 62]. And in method B, electrostatic and magnetostatic experimental facts, variable electric and magnetic fields are consistently analyzed and Maxwell's equations are derived [15, 53, 61].

Therefore, we will further consider these two methods as one and call it traditional, which is based on the generalization of the named experimental laws. However, there is another approach, namely:

C. In the textbook [49], Maxwell's equations are simply formulated as postulates, the validity of which is confirmed by the consequences obtained when applying them to describe specific electrodynamic situations.

This method may be the best when studying electrodynamics by mathematics students. At the same time, the process of substantiating, proving any physical law or relation is extremely important for revealing the content of this law, for forming the physical style of thinking of students and for understanding the dialectic of empirical and theoretical in the structure of physical knowledge. Therefore, we will not consider the method of postulating Maxwell's equations further, considering it unacceptable when teaching electrodynamics in the professional training of physics teachers.

In the methodological literature, another method is also used.

D. The method based on the principle of least action (PLA). It consists of the following. Based on general physical principles, the action function S is constructed (or even postulated) for a system consisting of an electromagnetic field and charged particles in this field, and then using it and the principle of least action, the equation of motion of a charged particle and the equation of the electromagnetic field, Maxwell's equations, are obtained [3, 41, 42, 48].

In our opinion, this method of deriving Maxwell's equations is abstract, formal and to some extent divorced from those physical ideas that are formed in physics students of pedagogical universities (detailed physical interpretation of electrodynamic phenomena, connection with initial definitions of physical quantities, visibility, professionally significant issues that have an output to future pedagogical activity and make it possible to adapt the received information to the school physics course).

Unlike this, we propose a methodology for teaching electrodynamics and a method for deriving Maxwell's equations based on a much smaller number of initial provisions than in the methods mentioned above.

That is, the method of deriving Maxwell's equations in the framework of the proposed methodology for teaching electrodynamics is based on using only Coulomb's law and special relativity theory. Starting from a fairly small system of equations, this method makes it possible to describe the electromagnetic field of a charged particle moving uniformly with an arbitrary velocity. Analysis of the properties of the electromagnetic field of a uniformly moving charged particle makes it possible to derive the system of Maxwell's equations in vacuum and explain and describe the main electromagnetic phenomena. In addition, within the framework of this methodology for teaching electrodynamics, it is possible to provide a physical explanation of a number of phenomena that are not paid attention to in the traditional methodology, and which, in our opinion, enable future physics teachers to understand the essence and content of electromagnetic field theory.

The proposed methodological system for teaching electrodynamics consistently uses the provisions of special relativity theory and differs in this from the traditional methodology for teaching electrodynamics. In addition, this methodology has a number of significant advantages over existing ones, which will be discussed in detail below.

The scientific and methodological foundations for studying special relativity theory and the structure of its presentation in pedagogical universities are analyzed in the author's monograph [33] and therefore, in order to compare both the positive and negative aspects of each of the methods of teaching electrodynamics, we will move on to describing the two methods mentioned above (A and D) for obtaining Maxwell's equations and a brief overview of the results in the context of professional training of physics teachers.

Thus, the traditional method of deriving Maxwell's equations (approaches A and B) is the most acceptable when studying electrodynamics from the point of view of both the principle of continuity in teaching and the direct connection with experimental data. It also makes it possible, to some extent, to familiarize students with the methods and methodology of obtaining scientific results. On the other hand, the process of scientific discovery itself (namely, how a fundamental physical law is obtained or formulated based on experimental facts) is instructive and interesting. This way of substantiation can also increase the interest of students and pupils in physics. But at the same time, students may get the impression that the fundamental equations and principles of physics are logically and consistently derived from experimental facts.

At the same time, it is known that there is no direct and logical path from experimental facts to a theoretical principle. And this important point in the methodology of scientific cognition and learning especially concerns the relationship between the "fundamental experimental laws of electrodynamics" and Maxwell's equations. "Maxwell's equations are an example of a fundamental law that was clearly guessed, and not "derived" in the rigorous sense of the word, from experimental data," is noted in the review article by Shapiro [59], devoted to the history of the discovery of Maxwell's equations.

In the author's monograph and textbooks [28, 30, 31, 33], the structure of teaching classical electrodynamics according to the traditional methodology and based on the principle of least action is considered in sufficient detail.

In particular, the structure of the content component of teaching classical and relativistic electrodynamics according to the traditional methodology can be presented as shown in figure 1.

We will focus only on the obvious conclusions that follow from the analysis presented in [33, 35].

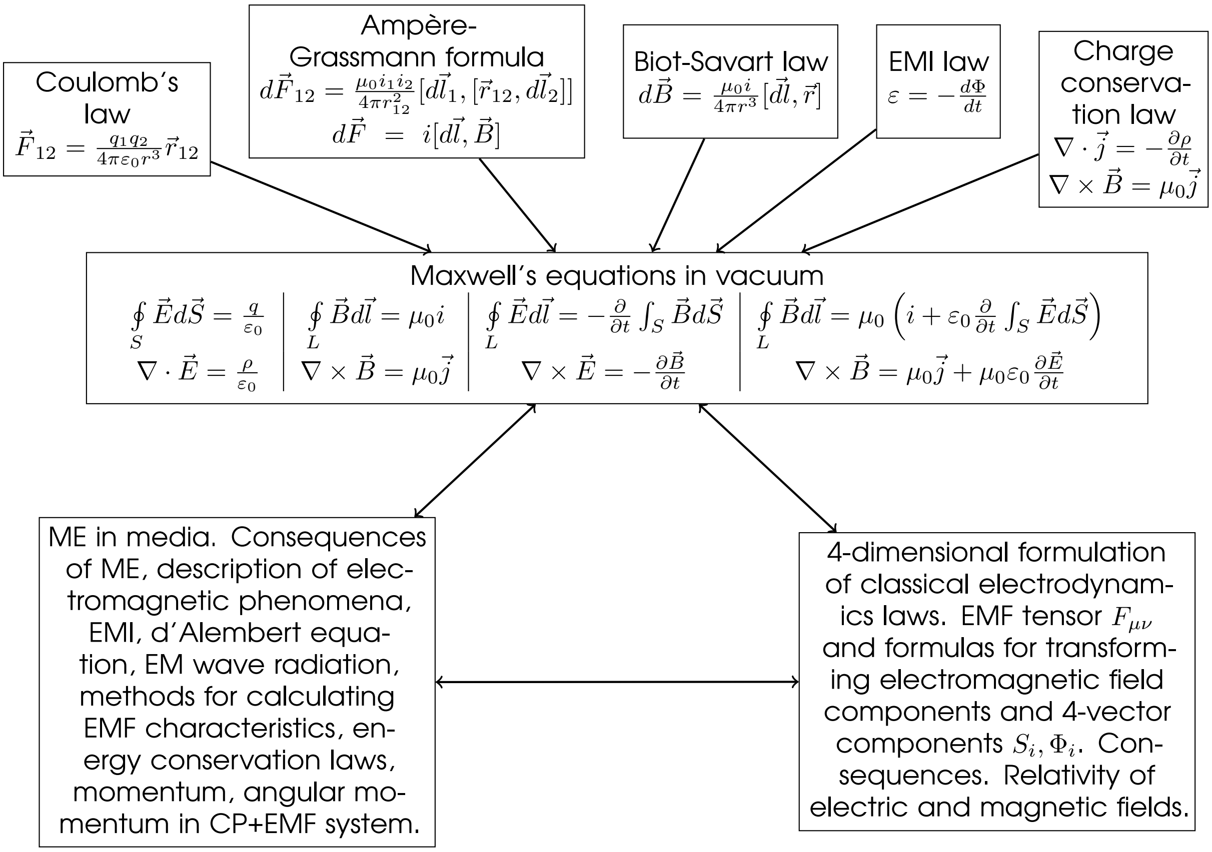

Figure 1: Structure of the content component for teaching electrodynamics using the traditional methodology.

From the above concise analysis of teaching electrodynamics according to the traditional methodology and based on the principle of least action, the following generalizations follow:

1. Within the framework of our analysis of teaching classical electrodynamics, for the first time we presented a description of the method of deriving Maxwell's equations based on PLA (for more details, see [33, p. 51-64]) in the SI system of units.
2. It turned out that for the consistent derivation of Maxwell's equations based on PLA, it is necessary to postulate:
   a) the form of the 4-dimensional potential of the electromagnetic field;
   b) the form of the action for a charged particle in an electromagnetic field;
   c) the validity of the principle of least action for a charged particle in an electromagnetic field.

   In addition, it is necessary to introduce additional concepts and quantities that are not easy for students to master: interval, 4-dimensional potential, 4-dimensional radius vector, action function.
3. Based on PLA, the first pair of Maxwell's equations is formally easily derived:

$$\nabla \cdot \mathbf{B} = 0 \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2}$$

   But at the same time, the connection with the initial definitions of physical quantities is lost, in particular with the concepts of electric field strength

$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$ and magnetic field induction $\mathbf{B} = \nabla \times \mathbf{A}$, which are introduced rather abstractly from the equation of motion of a charged particle in an electromagnetic field (3).

$$\frac{d\vec{p}}{dt} = -q\frac{\partial \vec{A}}{\partial t} - q \cdot \nabla\varphi + q[\vec{v}, \nabla \times \vec{A}] \tag{3}$$

4. It turns out that the construction of the theory and the study of electrodynamics based on PLA should be done with maximum approximation to both experimental facts and the gradual introduction of physical quantities in an extremely formal way.

At the same time, it is necessary to constantly trace the compatibility of the results obtained using PLA with Maxwell's equations, *which are obtained in another way, or are considered as a result of generalization of experimental facts* in this approach.

That is, within the framework of the method based on PLA, it is impossible in principle to consistently derive both Maxwell's equations and other basic laws of electrodynamics *without reference to Maxwell's equations already known from other considerations (not based on PLA).* It is necessary to constantly rely on experiment and on Maxwell's equations that are already known in advance (as if a priori).

Thus, in order to realize the possibilities of PLA in the context of obtaining Maxwell's equations, it is necessary, in addition to the listed postulates, *to also have ready (postulated independently of PLA "truths" in the form of Maxwell's equations themselves) Maxwell's equations* in order to find the necessary coefficients (see section 1.1.2.4 in [33]).

In addition, the method itself, although general and consistent with special relativity theory, draws information from experiment. This makes it, in our opinion, somewhat inconsistent.

5. The analyzed approach represents a long, abstract, formal substantiation, with the introduction of a significant number of 4-dimensional quantities, the content and properties of which are perceived by students with difficulty and unconvincingly.

We also have a rather long way to substantiate the expression for the action function of the electromagnetic field itself in the absence of a charged particle. But as a payment for this abstractness, we have, basically, a holistic, systemic, in Einstein's understanding, approach to the study of electrodynamics.

On this path, a number of extremely important results are obtained, such as: the equation of motion of a charged particle in 4-dimensional form, the electromagnetic field tensor, the field equations in 4-dimensional form [3, 42] and the second pair of Maxwell's equations in vacuum and medium [42, 58].

6. When studying the electromagnetic field in this way through varying the action function, many misunderstandings remain among students – why the action function for a charged particle in an electromagnetic field is chosen in this particular form and not in another, why when varying the action function the 4-dimensional current should be left unchanged, why when differentiating the Lagrangian function the velocity of motion of the charged particle should be considered unchanged, etc. Obviously, for reasons of compatibility of the theoretical consequences obtained on the basis of PLA with experimental facts.

In our opinion, such a path is unjustified when teaching electrodynamics in pedagogical universities, because it does not give a detailed physical interpretation of the electrodynamic phenomena being studied.

The disadvantage of this methodology is also that it is necessary to introduce rather formally the concepts of vector and scalar potentials of the electromagnetic field as components of the 4-dimensional potential. All this together gives the impression of something artificial and somewhat divorced from experiment.

At the same time, it should be noted that the derivation of the law of electromagnetic induction based on PLA in the form $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ after the equations of motion in the form of Lagrange equations are obtained is a certain positive moment of this method, since in traditional methods the initial fundamental experimental formulation of the law of electromagnetic induction is, in our opinion unjustifiably, the formula $\varepsilon = -\frac{d\Phi}{dt}$.

These two approaches in substantiating the first pair of Maxwell's equations reflect a characteristic trend in the history of the formation and development of electromagnetic field theory. We can even say that we have a problematic methodological situation. Namely: which quantities most adequately describe the state of an electrodynamic system – field strengths or field potentials.

7. The law of charge conservation when applying PLA follows as a consequence of the obtained Maxwell's equations. In the traditional methodology, the law of charge conservation plays an independent role, for example, as an element of contradiction with Maxwell's equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$.

Within the framework of the traditional method for deriving Maxwell's equations, electromagnetic potentials are introduced as one of the ways to solve them. They are introduced in order to present Maxwell's equations in the form of d'Alembert equations in terms of these quantities.

And in the method of deriving Maxwell's equations based on PLA, expressions for electromagnetic potentials in the form of a 4-dimensional potential are simply postulated and are initial.

The obtained generalizations highlighted the positive and negative aspects of using PLA in the study of classical electrodynamics and generally made it possible to consider that despite the presence of significant methodological advantages, such an approach is not favorable in teaching students of pedagogical universities.

3. Shortcomings and contradictions in methods of teaching electrodynamics

In the textbooks by Savel'ev [57], Sivuhin [60] the methodology for studying the magnetic field of steady currents is based on the expression for the magnetic field of a slowly moving charged particle (which is substantiated with the help of some plausible reasoning, but which at the same time "have no probative force" [57, p. 113]) and the superposition principle [57, p. 116].

And yet, despite the understanding that the Biot-Savart law

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \mathbf{r}}{r^3} \tag{4}$$

is fundamentally inaccessible to experimental verification, in the vast majority of modern educational and methodological manuals it is interpreted as experimental.

But analysis of the essence of this law and historical sources indicates the incorrectness of such an approach in studying and interpreting the relationship formulated in law (4) [19, 20, 33].

In addition, our studies have shown that in some cases the application of the Biot-Savart law (4) leads to results that contradict the basic provisions of relativistic electrodynamics [26, 33–35].

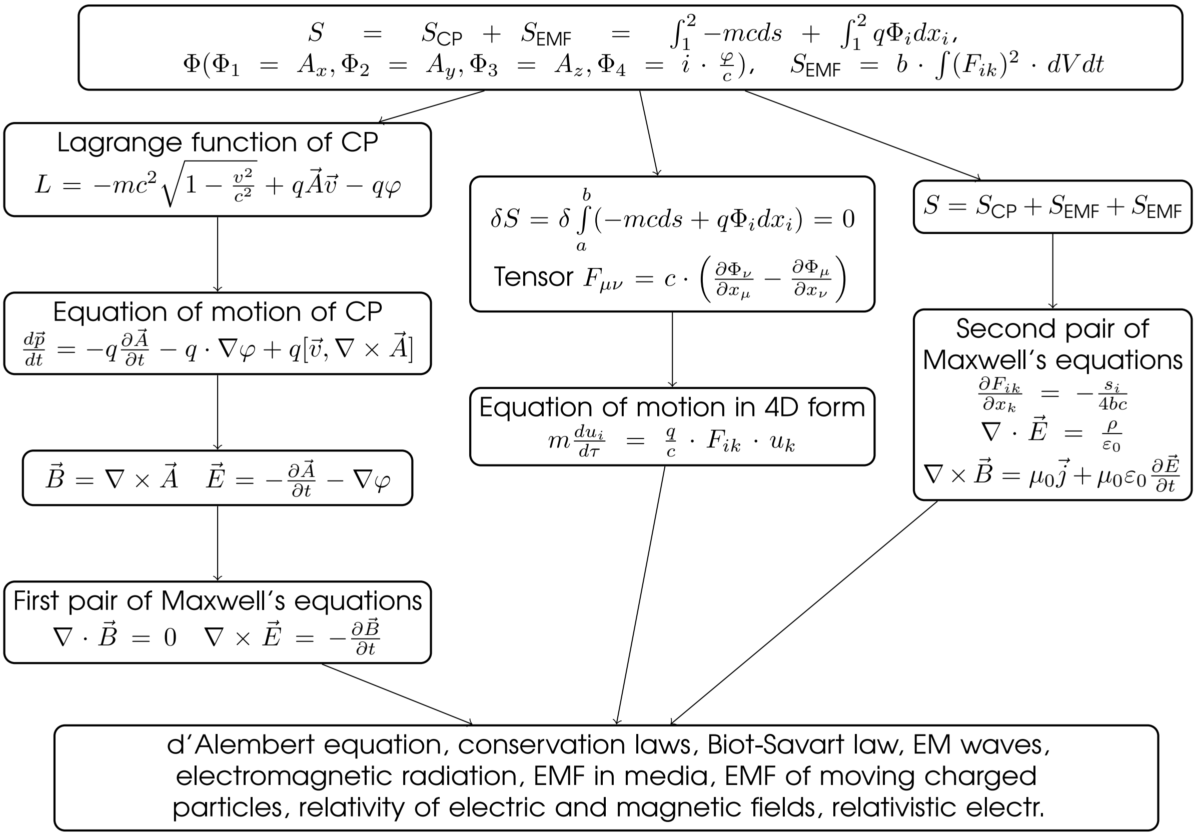

Figure 2: Structure of the content component of teaching electrodynamics based on the principle of least action.

That is, the application of the expression for the magnetic field induction of a moving charged particle, $\mathbf{B} = \frac{1}{c^2}[\mathbf{v} \times \mathbf{E}]$ (which can be considered as a consequence of the Biot-Savart law (4)) and $\mathbf{E} = \frac{q}{4\pi\varepsilon_0}\frac{\mathbf{r}}{r^3}$ for the analysis of the same electrodynamic problem leads to conclusions that contradict the principle of relativity.

Indeed, let us consider a thought experiment, the scheme of which is shown in figure 3. A charged particle moves with constant velocity $\mathbf{v}$ in the plane of contour L.

According to the traditional interpretation of the phenomenon of electromagnetic

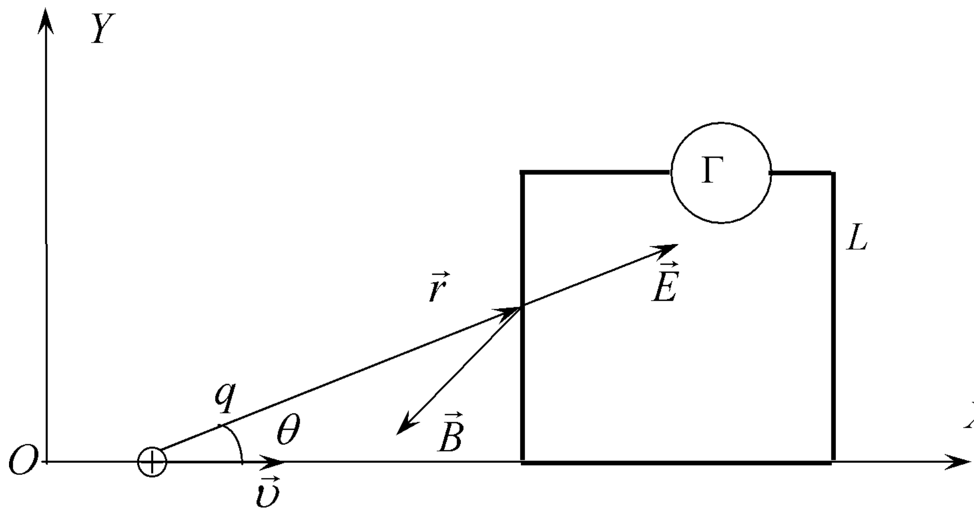

Figure 3: Scheme of a thought experiment to determine the EMF of induction in contour L.

induction in the contour, due to the change over time of the magnetic field induction at each point of the flat surface bounded by contour L (and hence the change in magnetic flux over time), an EMF of induction will arise.

But such a conclusion contradicts the principle of relativity: when transitioning to a reference frame associated with the charged particle, the contour will move in a centrally symmetric Coulomb field. Such a field is potential, so there are no physical reasons that could generate an EMF in contour L.

Similarly, in the problem shown in figure 4, the use of the classical Biot-Savart law leads to a result that contradicts the principle of relativity.

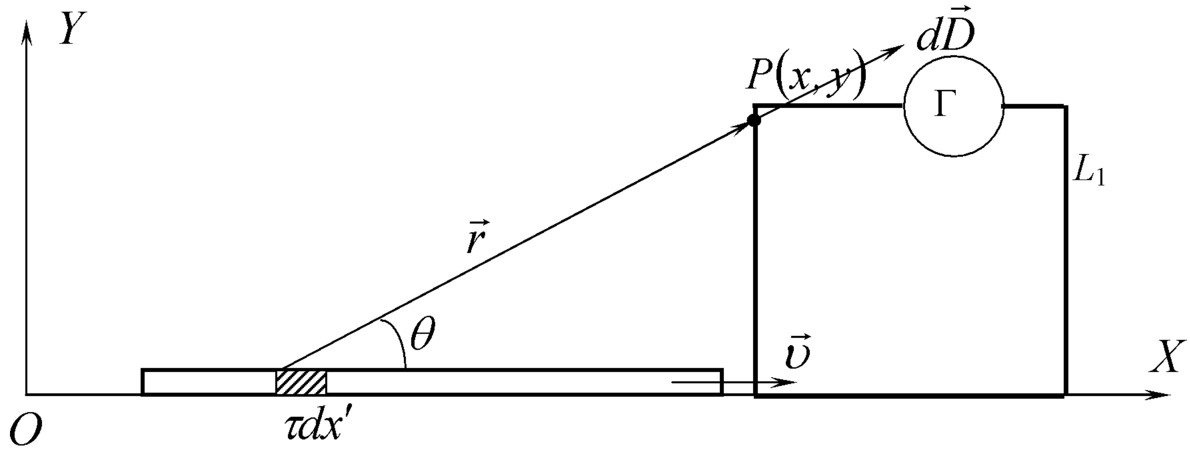

Figure 4: When a charged thread moves with velocity $\mathbf{v}$, there is no EMF in contour L.

According to the Biot-Savart law (4) and the traditional interpretation of the phenomenon of electromagnetic induction, one should expect the appearance of a certain EMF of induction in contour L_1 (see the problems shown in figure 5 and figure 6). Such a conclusion follows from the following considerations.

Each of the conduction electrons moving with drift velocity in the conductors of the electric circuit creates a variable magnetic field at each point of the flat surface bounded by contour L_1. This variable magnetic field, in turn, generates a vortex electric field ($\nabla \times \mathbf{E} \neq 0$) at each point of the surface. The integral effect should manifest itself in the appearance of an induction current in contour L_1. But experiments show that no EMF arises in contour L_1.

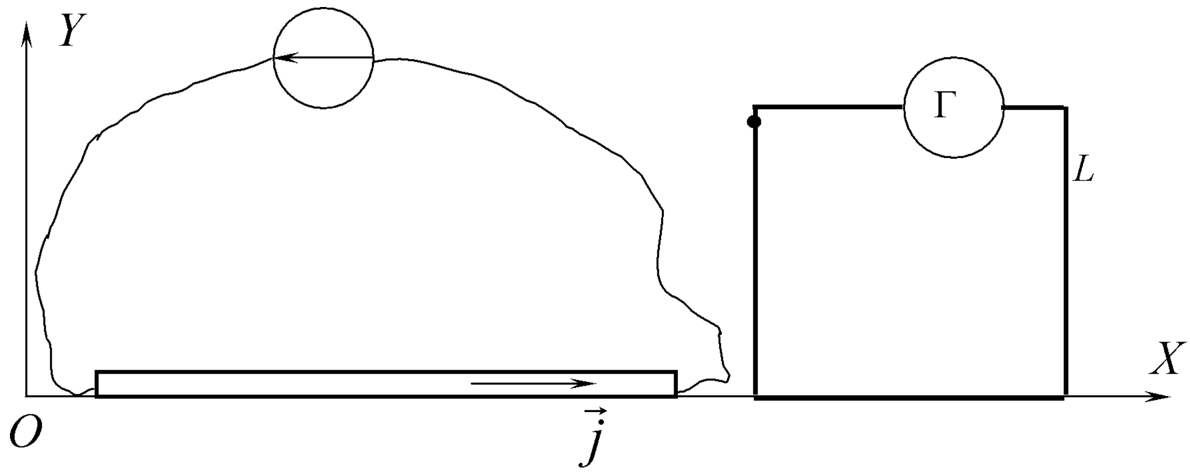

Figure 5: In a wire contour L located in the vicinity of an infinitely long conductor with a steady current, there is no EMF.

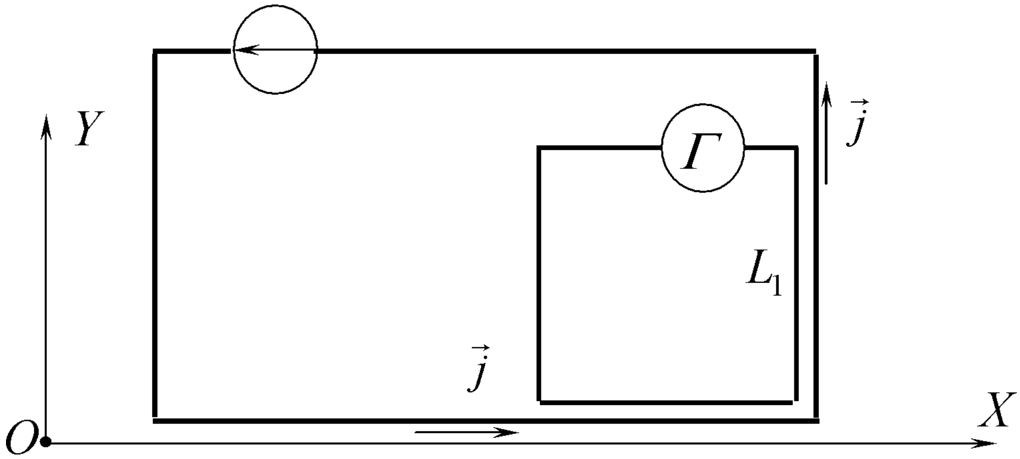

Figure 6: In a conducting contour L_1 located next to a steady current circuit, there is no EMF.

This means that the application of the Biot-Savart law and its equivalent – the expression for the magnetic field generated by a separate moving charged particle – $\mathbf{B}_1 = \frac{1}{c^2}[\mathbf{v} \times \mathbf{E}_1] = \frac{\mu_0}{4\pi}\frac{q\mathbf{v}\times\mathbf{r}}{r^3}$ in such problems gives a result that contradicts physical reality.

Thus, any simultaneous use of these formulas (or simultaneous application of the classical Biot-Savart law and the expression for the electric field strength) leads to fundamental errors.

Therefore, the traditional methodology for teaching electrodynamics, which is based, in particular, on the classical Biot-Savart law, is unable to explain the absence of EMF in the contours in figures 3, 4, 5, 6. At the same time, in standard textbooks and scientific-methodological publications, these two formulas are used together, which is a gross physical error.

Obviously, a change in the dogmatic methodology of teaching electrodynamics is needed.

So, it is clearly seen that there is a need to develop theoretical and methodological foundations for teaching electrodynamics, which would be based on consistently relativistic principles and would not have the above-mentioned shortcomings.

Thus, we can make the following generalizations.

1. The method of deriving Maxwell's equations and methods of teaching classical and relativistic electrodynamics based on the principle of least action in the professional training of physics teachers within the framework of basic higher education in pedagogical universities should be considered too difficult to understand due to their abstractness, formality and some detachment from those physical ideas that are formed in physics students of pedagogical universities. In particular, we are talking about the content of electric and magnetic field induction, as well as their definition through $\mathbf{A}$ and ϕ as a consequence of the equation of motion.
   This methodology, in addition, requires the preliminary introduction of rather formal concepts of vector and scalar potentials of the electromagnetic field as components of the 4-dimensional potential.
   All this together gives the impression of something artificial and somewhat divorced from experiment. Such an approach cannot be considered justified when teaching electrodynamics because it does not give a detailed physical interpretation of the electrodynamic phenomena being studied.
2. Better for assimilation and understanding of Maxwell's theory, in our opinion, is

the approach in deriving Maxwell's equations based on experimental laws. Since it is more visual, unlike the approach based on the principle of least action, it contains much fewer elements of vector algebra.
In addition, within the framework of this method, it is possible to explain more accessibly the physical essence of such concepts as electromagnetic field, electromagnetic field strength, induction, etc.

3. At the same time, the traditional methodology for teaching electrodynamics in pedagogical universities is characterized by a number of shortcomings. First of all, in our opinion, it has an electrical engineering character. Excessive generalization of empirical facts, inconsistency of presentation, neglect of relativistic corrections in substantiating phenomena that directly belong to the field of special relativity theory, leads to some alienation of the content of electrodynamics as an academic discipline from relativistic physics.
4. The characteristic tendency of the development of modern physics also does not find adequate reflection in students' study of electrodynamics: relying on a small number of basic principles to explain the entire set of physical phenomena and laws of this section of physics.
Therefore, when studying electrodynamics according to the traditional methodology, the principle of fundamentalization is not implemented.
5. When analyzing the educational material, the need to correct "traditional" approaches to mastering basic concepts and phenomena of electrodynamics by students was revealed, in particular:
    - The law of electromagnetic induction needs such a generalization that its local form would reflect two physical reasons that underlie the phenomenon of electromagnetic induction.
    - The Biot-Savart law and Coulomb's law should not be used simultaneously in the analysis of electromagnetic phenomena, because this leads to fundamental errors and contradictions.
    - The laws of Biot-Savart, Ampère-Grassmann, in turn, themselves need substantiation, since they are not purely experimental.

   The traditional methodology and the methodology based on PLA provide a formal and unconvincing substantiation of Maxwell's equation, overloaded with a large number of "independent fundamental experimental facts".
6. It is shown that in the structure of teaching classical electrodynamics according to the traditional methodology, a number of contradictions arise between theoretical substantiation and experimental factors. First of all, it concerns the physical explanation of the phenomenon of the occurrence of the magnetic field of steady currents.

Indeed, in textbooks on electrodynamics for both higher education and secondary schools, the question of the mechanism of the occurrence of the magnetic field of steady currents is not discussed at all. The discussion and physical explanation is reduced to phrases like "electric current is accompanied by a magnetic field", "a magnetic field is associated with the motion of charged particles", "around moving charges (currents) there is a magnetic field".

As is known, the magnetic field in space outside steady currents is a potential field ($\nabla \times \mathbf{B} = 0$). This conclusion is not consistent with the idea of the magnetic field of currents as a vortex field, because this is possible under the condition that $\nabla \times \mathbf{B} \neq 0$.

But calculations of the magnetic field induction in space around a conductor with a steady current using the equation $\nabla \times \mathbf{B} = 0$ and its equivalent $\nabla^2 \mathbf{A} = 0$, given in section 6, give results that coincide with experimental ones.

Thus, the analysis showed that with the "traditional" approach to studying electrodynamics in a pedagogical university, as well as with the methodology based on PLA, a number of unresolved scientific and methodological problems, contradictions and inconsistencies arise. Therefore, the search for an optimal methodology for deriving Maxwell's equations and methods of teaching classical electrodynamics is an urgent scientific and methodological problem. The solution to this problem is shown in the following sections of this paper.

4. Electric field of a moving charged particle and its properties

The study of the mechanisms of generation of the magnetic field, explanation of the relativistic nature of the magnetic field should begin with an analysis of the properties of the electric field of a moving charged particle (CP).

Charged particles that move relative to the laboratory reference frame interact differently than stationary ones. This conclusion follows from the analysis of the experimental fact regarding the interaction of two parallel currents.

But obviously, the resulting interaction of conductors with currents is due to an additional (*non-electrical*) interaction between sets of moving charged particles (two precessions of CP).

Attention should also be paid to the fact that on the basis of Newtonian mechanics it is impossible to explain the origin of magnetic forces. In Newtonian mechanics, force is a function of the distance between interacting bodies and their relative velocities. But the distances between conduction electrons in two parallel linear conductors with currents do not change; their velocity relative to each other remains zero. It would seem that no auxiliary forces should arise. That is, according to the basic provisions of classical mechanics, the interaction forces between uniformly moving CPs should not differ from the interaction forces between stationary CPs.

But such a conclusion contradicts reality. Experience shows that when electrons move in parallel linear conductors, forces arise between them that were not present in the absence of current. This demonstrates the limitations of Newtonian mechanics and the unsuitability of its representations for interpreting magnetic interaction ([51, p. 103], [54]). It turns out that on the basis of classical physics concepts we are unable to explain the experimental manifestations of magnetic interaction.

Thus, to describe and understand the interaction between moving charged particles (and in fact the interaction between current elements and conductors with currents) it is necessary to describe on the basis of relativity theory the interaction of two moving charged particles.

But first we need to find an expression for the electric field strength of a single point charged particle moving at constant velocity.

Using Coulomb's law and the provisions of special relativity theory, we can find an expression for the electric field strength of a uniformly moving charged particle [31–33, 56, 63]:

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{1-V^2/c^2}{[(x-Vt)^2+(y^2+z^2)(1-V^2/c^2)]}\mathbf{r} = \frac{q}{4\pi\varepsilon_0}\frac{f(\beta,\theta)}{r^3}\mathbf{r} \tag{5}$$

where $\mathbf{r} = (x-Vt)\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ is the radius vector drawn from the instantaneous position of the CP to the field point, r is the distance of the field point from the instantaneous position of the CP, θ is the angle between the direction of motion of the CP (velocity vector $\mathbf{v}$ of the CP) and the radius vector drawn from the instantaneous position of the CP to a given point in space (see figure 7), $\beta = V/c$, $f(\beta,\theta) = \frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}$.

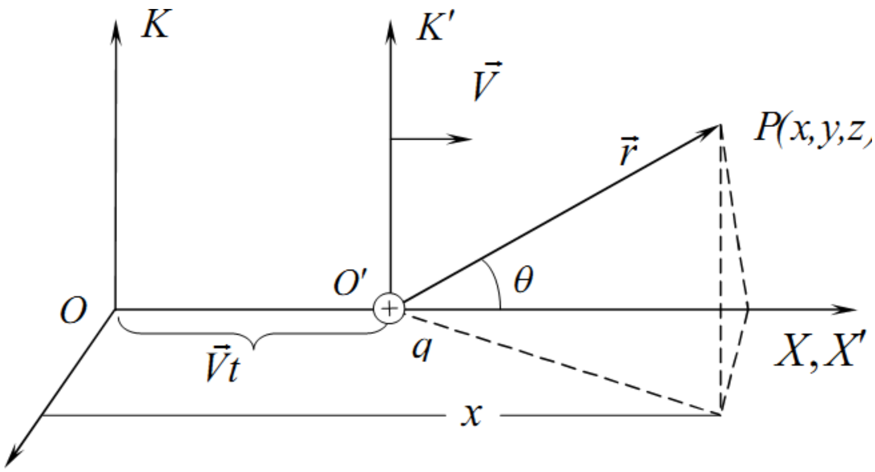

Figure 7: Illustration for the formula of the electric field strength of a moving CP.

In the available educational and methodological publications, the fundamental difference between the electric field of a moving CP and the electric field of a stationary CP is not analyzed, moreover, no attention is paid to it. Such an approach can lead at least to errors in calculations and incorrect conclusions.

In the author's publications [31–33], we made a comparative analysis of ways to substantiate formula (5) and for the first time analyzed in detail the properties of the electric field of a moving CP and developed a computer model of the electromagnetic field of a uniformly moving charged particle (see section 9).

As will be seen from the following analysis, even a slight difference in the electric field of a moving CP (at $v \ll c$) from the electric field of a stationary CP is of fundamental nature.

5. Magnetic interaction and STR. Biot-Savart law in relativistic form

Our research on the study of electrodynamics based on STR, methods of forming the concept of "magnetic field" and explaining the relativistic nature of the magnetic field were initiated by the use of relativistic ideas in the study of physics proposed in the textbook [17]. Only after completing the main research on this topic, the author was able to familiarize himself with the dissertation work of Pinskij [54].

We believe that our approach to explaining the relativistic nature of the magnetic field is more general than in [54], since in our model the radius vector connecting two CPs is *oriented arbitrarily* relative to the velocity vector $\mathbf{V}$. And this opens up wider possibilities for us in interpreting many phenomena of electrodynamics. And in works [17, 54] the moving CPs are located on the OY axis.

In addition, in works of Pinskij [54], only one single case (out of many possible others) is considered when at the moment $t = 0$ the origins of the coordinate systems of two reference frames coincide. This simplification does not allow, in our opinion, to obtain in works [17, 54] the functional dependence $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$ of a moving CP and, moreover, in work [54] only the transverse components of the field vectors $\mathbf{E}$ and $\mathbf{B}$ are considered.

In our study, we formulate the problematic situation that arises when describing the interaction of two CPs in a completely different way. Firstly, the radius vector that determines the position of one CP relative to another CP is oriented arbitrarily relative to the vector $\mathbf{V}$ – the velocity of motion of the two CPs. And in work [54] this radius vector is perpendicular to $\mathbf{V}$ (that is, both CPs lie in a plane perpendicular to $\mathbf{V}$).

Secondly, we analyze in detail the interaction of two moving CPs based on STR methods [33, p. 23-27], while in [54] the expression $F_y = F'_y(1 - V^2/c^2)^{1/2}$ is reduced by algebraic transformations to

$$F_y = \frac{F'_y\sqrt{1 - V^2/c^2}\sqrt{1 - V^2/c^2}}{\sqrt{1 - V^2/c^2}} = \frac{1}{\sqrt{1 - V^2/c^2}}\left(F'_y - F'_y\frac{V^2}{c^2}\right) \tag{6}$$

The term $(-F'_y V^2/c^2(1 - V^2/c^2)^{-1/2})$ in dissertation by Pinskij [54] is called (just like that, without justification) the Lorentz force.

Since electric current in conductors is the directional motion of conduction electrons, therefore, from observations of the interaction of parallel conductors with currents, as already noted, an obvious conclusion follows: between two precessions of moving electrons there is an additional force that was not present between stationary electrons.

Therefore, we can find out the nature of this force by analyzing the interaction of two charged particles moving parallel to each other with the same or different velocities.

Therefore, the analysis and understanding of magnetic interaction should be based on the analysis of the interaction of two moving CPs. For simplicity and clarity of analysis, let us consider the interaction of two CPs moving parallel to each other at constant velocities.

Note that for a correct consideration of any phenomenon from the point of view of reference frames that are in relative motion, one should rely on the principle of relativity. That is, in the process of establishing the relationship between an arbitrary physical quantity measured in different reference frames, one should use only the methods of special relativity theory. After all, as Einstein emphasized, only theory can say what is actually measured in an experiment.

Thus, we will analyze the interaction of two moving CPs taking into account the exact relationships between physical quantities characterizing the motion and interaction of CPs [23, 33].

Example 1.1. Let in the reference frame K' in the plane Y'X' there are two stationary charged particles (hereinafter – protons), the charge values of which are q_1 and q_2, and the distance between them is r' (figure 8). The reference frame K' moves at a constant velocity $\mathbf{V} = const$ relative to the laboratory reference frame K along the OX axis. Describe the interaction between the charged particles.

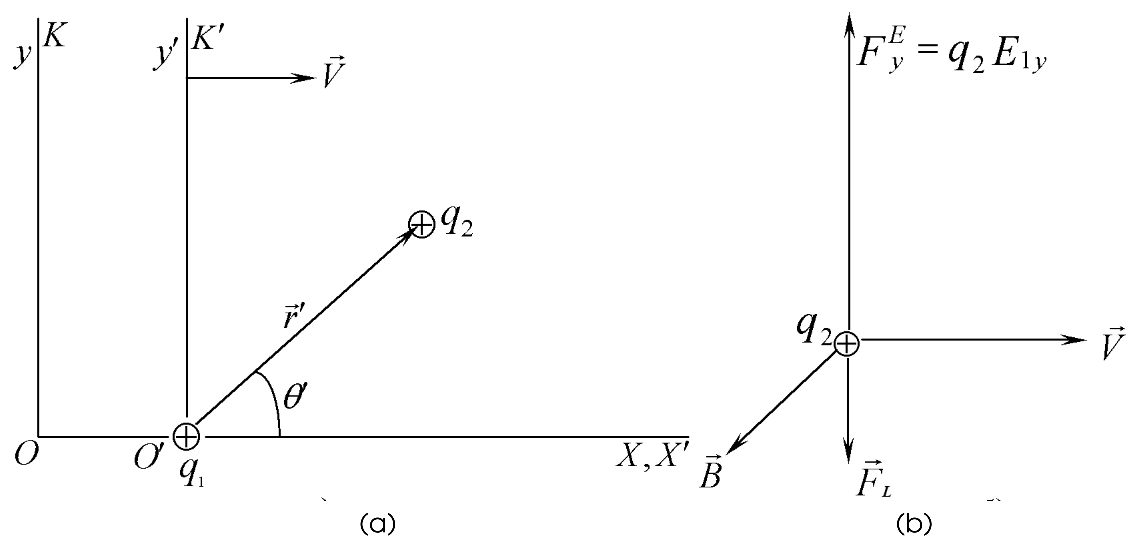

Figure 8: Interaction of two charged particles moving at constant velocity $\mathbf{V}$ relative to the LRF (a) and the transverse component of the force acting on the second CP (b).

Solution: In the reference frame K' there is only electric interaction between the CPs. That is, a force $\mathbf{F}_2' = q_2\mathbf{E}_1'$ acts on the second CP (with charge value q_2), where $\mathbf{E}_1'$ is the electric field strength created by the first CP at the location point of charge q_2:

$$\mathbf{E}_1' = \frac{q_1}{4\pi\varepsilon_0}\frac{\mathbf{r}'}{r'^3}$$

But in the reference frame K, the electric field strength is determined by formula (5).

It is easy to verify, using the Lorentz transformations, that the longitudinal component of the interaction force between charges q_1 and q_2 does not change when transitioning from the reference frame K' to the reference frame K.

Now let's move on to the analysis of the transverse component of the interaction between these CPs. As can be seen from the expression for the electric field strength of a moving CP (5), the transverse component of the electric interaction force between CPs in the reference frame K increases, that is:

$$F_y^E = q_2E_{1y} > q_2E_{1y}'$$

where E_{1y}, E_{1y}' are the transverse components of the electric field strength created by the first CP at the location point of the second CP in the reference frames K and K' respectively.

As is known, the principle of relativity states that physical phenomena proceed in the same way (under identical initial conditions) in all inertial reference frames. In other words, the mathematical form of the basic laws of physics should not change when transitioning from one inertial reference frame to another: the equations of physics should be Lorentz covariant; at the same time, the spatial and temporal coordinates (x, y, z, t) of any event in the reference frames K and K' are related by the Lorentz transformations.

For the basic equations of physics to have the same mathematical form in the reference frames K and K', the projections of force (in particular) on the coordinate axes should transform according to formulas (20) of monograph [33].

Since the CPs are stationary in the reference frame K', then $v_x' = 0$, in our case (figure 8) we have for the y-component of the interaction force between moving CPs in the reference frame K:

$$F_y = \frac{F_y'\sqrt{1-\beta^2}}{1+Vv_x'/c^2} = F_y'\sqrt{1-V^2/c^2}$$

Then $F_y < F_y'$, because $\sqrt{1-V^2/c^2} < 1$.

That is, despite the fact that the transverse component of the electric interaction force between CPs in the reference frame K increases, the principle of relativity requires that the total transverse component of the interaction force between CPs in our problem be smaller than the transverse component of the interaction force between CPs in the reference frame K', and equal to

$$F_y = F_y'\sqrt{1-V^2/c^2} \tag{7}$$

Thus, we have: the electric force of interaction in the direction of the OY axis increases in the reference frame K, and the principle of relativity requires that the total force of interaction between moving CPs decrease according to formula (7). This means only one thing – in the reference frame K an additional interaction appeared between moving charged particles, which was not present in the system K'.

In other words, we must assume the emergence of such a transverse force of interaction between moving CPs in the reference frame K, the existence of which is necessary for the fulfillment of the principle of relativity. When the CPs are stationary in the reference frame K', there is only an electric force $\mathbf{F}' = q_2\mathbf{E}'_1$ between them, when they move at velocity $\mathbf{V}$ relative to the reference frame K, we are forced to assume the appearance (obviously, due to the motion of the CPs) of some additional transverse force acting on the second CP.

Let us denote this, as yet unknown, force $\mathbf{F}_L$. This force reduces the electric component of the force acting on the second CP, F_y^E, to the value $F_y = F'_y\sqrt{1 - V^2/c^2}$, which is *required by the principle of relativity*.

Thus, we can write

$$q_2E_{1Y} - F_L = F'_Y\sqrt{1 - V^2/c^2} \tag{8}$$

The absolute value of the force is found from (8)

$$F_L = \frac{q_1q_2(1 - V^2/c^2)\sin\theta \cdot V^2}{4\pi\varepsilon_0 r^2c^2(1 - V^2/c^2)^{3/2}} = \frac{q_2V^2}{c^2}E_1\sin\theta \tag{9}$$

This force is always perpendicular to the velocity of motion of CP q_2 (see figure 8b) and parallel to the OY axis, wherever charge q_2 is located in the XOY plane.

Moreover, the transverse component of the force acting on q_2 should be interpreted by us as a force acting on the second moving CP in some physical field, the existence and appearance of which is required by the principle of relativity. The properties and characteristics of this field follow from (8) and (9).

Taking into account the spatial arrangement of forces and the velocity of motion $\mathbf{V}$ of charged particles, as well as the invariance of the longitudinal component of the electric field (and the invariance of the longitudinal component of the force acting on q_2), which is created by charge q_1, the expression in vector form of that force $\mathbf{F}_L$, whose existence is required by STR, must be written in the following form (figure 8)

$$\mathbf{F}_L = q_2\left[\mathbf{V}, \left[\frac{\mathbf{V}}{c^2}, \mathbf{E}_1\right]\right] \tag{10}$$

They say that $\mathbf{F}_L$ is the force acting on charge q_2, which moves at velocity $\mathbf{V}$ in a field characterized by the quantity $[\mathbf{V}/c^2, \mathbf{E}_1]$, and which, in turn, is created by the motion of the first CP, whose charge value is q_1. That is, (10) is presented in the form

$$\mathbf{F}_L = q_2[\mathbf{V}, \mathbf{B}_1] \tag{11}$$

This field is called magnetic, and the magnetic induction $\mathbf{B}_1$ of a moving CP is equal to

$$\mathbf{B}_1 = \frac{1}{c^2}[\mathbf{V}, \mathbf{E}_1] = \frac{\mu_0}{4\pi}\frac{q_1}{r^3}\frac{1 - V^2/c^2}{(1 - V^2\sin^2\theta/c^2)^{3/2}}[\mathbf{V}, \mathbf{r}] \tag{12}$$

If the velocities of charges q_1 and q_2 are different ($\mathbf{v}_1$ and $\mathbf{v}_2$ respectively), then generalizing (11), we obtain the force acting on a moving electrically charged particle q_2 in the field of a moving charge q_1

$$\mathbf{F}_L = q_2\left[\mathbf{v}_2, \left[\frac{\mathbf{v}_1}{c^2}, \mathbf{E}_1\right]\right] = q_2[\mathbf{v}_2, \mathbf{B}_1] \tag{13}$$

Obviously, when an electron moves, the induction of its magnetic field is determined by the formula

$$\mathbf{B}(\mathbf{r}, t) = -\frac{\mu_0}{4\pi}\frac{q_1}{r^3}\frac{1 - V^2/c^2}{(1 - V^2\sin^2\theta/c^2)^{3/2}}[\mathbf{V}, \mathbf{r}] \tag{14}$$

Thus, the above analysis of the interaction of two moving charged particles leads to the following results.

1. The need to introduce the concept of "magnetic field" is due to the requirements of relativity theory, namely: for the equations of motion of a material point to have the same form in the reference frames K and K', forces must transform according to certain formulas when transitioning from one reference frame to another, and taking into account the expression for the electric field strength of a moving CP (5) the additional force that ensures the fulfillment of these requirements is determined by
$$\mathbf{F}_L = q_2\left[\mathbf{V}, \left[\frac{\mathbf{V}}{c^2}, \mathbf{E}\right]\right] = q_2[\mathbf{V}, \mathbf{B}]$$
That is, the magnetic field with induction $\mathbf{B} = [\mathbf{V}/c^2, \mathbf{E}]$ is introduced in order to ensure the fulfillment of the principle of relativity in electrodynamics.
But at the same time, nothing can be said about the mechanism of occurrence, creation or generation of this magnetic field.
Only the necessity of the existence of such a field associated with a moving CP, which is characterized by properties (11), (12), follows from the requirements of STR.
2. Magnetic interaction is an essentially relativistic effect that arises at any velocity of motion of charged particles, even at $v \ll c$. But at the same time, the analysis should be carried out in accordance with exact relativistic relations, because if we do not take into account exact relativistic formulas (neglect somewhere the velocity of motion of CP v compared to the speed of light in vacuum c) we will not obtain magnetic interaction.
3. This approach makes it possible to immediately obtain an expression for the Lorentz force, (formulas (11) and (13)), acting on charge v_2 from the magnetic field generated by the motion of the first CP (charge value q_1).
And, as we can see, the Lorentz force arises not with "motion relative to the magnetic field", but with the motion of a charged particle in a magnetic field, because from the point of view of the reference frame K charges q_1 and q_2 are stationary relative to each other.
The substantiation of expression (11) for the Lorentz force in the general case will be obtained when one of the CPs moves in a magnetic field created not by the motion of another CP, but by arbitrary external sources of magnetic field (for example, a linear procession of other CPs).
4. The definition of the expression for the Ampère force can be obtained based on the consideration of a linear procession of CPs (for example, a conductor with current I) in a magnetic field $\mathbf{B}$, created by the motion of other CPs.
Since each of the conduction electrons that realize the current element acts on the Lorentz force, then for the Ampère force acting on the current element $Id\mathbf{l}$ in a magnetic field with induction $\mathbf{B}$ we find
$$d\mathbf{F} = -q[\mathbf{v}, \mathbf{B}]N = I[d\mathbf{l}, \mathbf{B}] \tag{15}$$
because $Id\mathbf{l} = -Nq\mathbf{v}$, where N is the number of electrons that realize the current element $Id\mathbf{l}$; $\mathbf{v}$ is the drift velocity of conduction electrons.
5. We come to the conclusion that at one and the same space-time point, the electric field strength and magnetic field induction of the electromagnetic field associated with the motion of CP are interdependent and related by the relation:
$$\mathbf{B}_1 = \frac{1}{c^2}[\mathbf{V}, \mathbf{E}_1] = \mu_0\varepsilon_0[\mathbf{V}, \mathbf{E}_1]$$
Generalizing the last expression, we can state that any electric field that "moves" at velocity $\mathbf{V}$ generates a magnetic field at the same space-time point, the magni-

tude and direction of which are determined by the formula:

$$\mathbf{B} = \left[\frac{\mathbf{V}}{c^2}, \mathbf{E}\right] \tag{16}$$

Analyzing this model in more detail, we can substantiate the transformation formulas for the components of the electromagnetic field when transitioning from one reference frame to another.

6. *We obtain the Biot-Savart law in relativistic form* using the expression (14) and the superposition principle for magnetic fields generated by individual electrons that are part of the current element $Id\mathbf{l}$:

$$d\mathbf{B} = N\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{I[d\mathbf{l},\mathbf{r}](1-\beta^2)}{r^3(1-\beta^2\sin^2\theta)^{3/2}} = \frac{\mu_0}{4\pi}\frac{I[d\mathbf{l},\mathbf{r}](1-\beta^2)}{((x-vt)^2+(y^2+z^2)(1-\beta^2)^{3/2}} \tag{17}$$

where I is the current strength, $\mathbf{r}$ is the radius vector drawn from the current element $Id\mathbf{l}$ to a given field point, $\mathbf{r} = (x-vt)\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $\beta = v/c$, v is the drift velocity of conduction electrons in the conductor with current I; c is the speed of light in vacuum, θ is the angle between $\mathbf{r}$ and the current element $Id\mathbf{l}$ [23, 26].
Obviously, for $\beta = v/c \ll 1$ the law (17) transitions to the usual Biot-Savart law (4)

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I[d\mathbf{l},\mathbf{r}]}{r^3} \tag{18}$$

Thus, we can state that law (17) – the Biot-Savart law in relativistic form, is more general and accurate. In any case, for uniform and rectilinear motion of charged particles, only it provides an adequate and correct description of magnetic interaction (see below and appendices A, B).

7. Based on the author's approach applied, it turns out that the contradictions that arise with the joint application of the classical Biot-Savart law and the expression for the electric field strength (see figures 3-6) are naturally refuted by using the Biot-Savart law derived above in relativistic form (17) and the expression for the electric field strength of a uniformly moving CP (5).
8. The magnetic induction vector is perpendicular to the electric field strength and the velocity of CP motion. Therefore, the field lines of the magnetic induction vector form a set of concentric circles lying in a plane perpendicular to $\mathbf{V}$, and whose centers are on the line of motion of the CP (see section 9).
By the way, this conclusion, which follows from the expressions for $\mathbf{E}$ and $\mathbf{B}$ of a moving CP, leads to the formulation of the "right-hand rule" for determining the direction of magnetic field lines of currents in the secondary school physics course.
9. Thus, the Biot-Savart law in relativistic form (17) follows as a consequence of Coulomb's law and the principle of relativity.

In other words, we have shown that the expression for the magnetic field induction created by a current element at an arbitrary velocity of motion of charge carriers is determined by law (17).

How accurately does law (17) correspond to reality? It is very difficult to verify (17) experimentally, especially since even the classical Biot-Savart law (18) is hardly verifiable by direct experiment.

In addition, in section 3 we have shown that when analyzing the electromagnetic field (which is created by uniformly moving charged particles) using the classical Biot-Savart law, we come to erroneous conclusions. These conclusions contradict both the fundamental principle of relativity and experimental observations.

At the same time, in the educational and methodological literature, it is considered that indirect confirmation of the classical Biot-Savart law (18) is the consistency of the results of magnetic field calculations based on it and the corresponding experimental facts. But there are other criteria. Let us compare the results of our analysis [19, 26] of some physical situations using the Biot-Savart law in classical and relativistic forms.

For this, let us use the Biot-Savart law in relativistic form to find the magnetic field induction created by: an infinitely long linear conductor with steady current (SCC) and a segment of a linear conductor with steady current i [33].

In section 3.3.1 of the monograph [33] it is shown that the magnetic field inductions created by an infinitely long linear conductor with steady current and a segment of a linear conductor with steady current i of length l give, respectively, the following results:

$$B = \frac{\mu_0 i}{2\pi R} \tag{19}$$

$$B = \frac{\mu_0 i}{4\pi R}\left(\frac{l - l_1}{\sqrt{(l-l_1)^2 + R^2(1-\beta^2)}} + \frac{l_1}{\sqrt{l_1^2 + R^2(1-\beta^2)}}\right) \tag{20}$$

We obtained expression (20), which differs from the well-known one, but for $\beta \ll 1$ equals it.

And if in the last formula we take $l - l_1 \gg R$, $l_1 \gg R$, then we get $B_z = \frac{\mu_0 i}{2\pi R}$ – the usual expression for the magnetic field induction of an infinitely long straight conductor with current i.

Further, when calculating the electric field strength and magnetic field induction of a uniformly charged thread moving at constant velocity, the results obtained (see below) based on (5) and based on the Biot-Savart law in relativistic form (17) coincide with those obtained using absolutely accurate formulas (*).

Indeed, let's consider the **Problem**: Let a uniformly charged thread move at velocity $\mathbf{V} = const$ along the OX axis of the reference frame K. The linear charge density on the thread in its own reference frame K' equals τ', length l'. Find the electric and magnetic fields at point A of the reference frame K, relative to which K' – system moves at velocity $\mathbf{V}$ (figure 9).

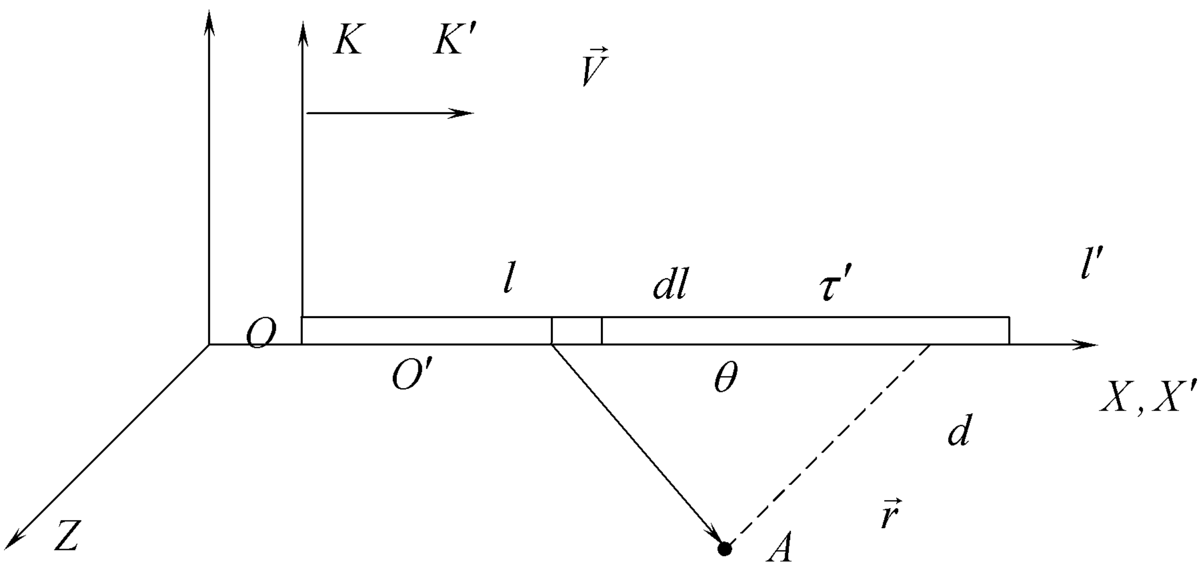

Figure 9: Calculation of electric field strength and magnetic field induction of a uniformly charged thread moving at velocity $\mathbf{V} = const$.

Solution: On the one hand, these fields can be found using the transformation formulas for the components of the electromagnetic field (*) when transitioning from the reference frame K' to the reference frame K

$$\begin{cases} E_z = \frac{1}{\sqrt{1-V^2/c^2}} \cdot (E'_z y - V B'_y) \\ B_y = \frac{1}{\sqrt{1-V^2/c^2}} \cdot (B'_y + \frac{V}{c^2} E'_z) \end{cases} \tag{*}$$

where E'_z, B'_y are the corresponding components of the electromagnetic field in the PRF K'.

$$B'_y = 0, E'_z = \frac{\tau'}{4\pi\varepsilon_0 d}\left(\frac{x'}{\sqrt{x'^2+d^2}} - \frac{x'-l'}{\sqrt{(x'-l')^2+d^2}}\right),$$

where x' is the x-coordinate of the field point in the reference frame K', d is the distance of field point A from the thread.

On the other hand, these quantities can be found using the superposition principle and expressions for fields dE_Z, dB_y, which are created by an elementary segment dl of a moving charged thread

$$dE_z = \frac{(1-\beta^2)\sin\theta}{4\pi\varepsilon_0 r^2(1-\beta^2\sin^2\theta)^{3/2}} dq = \frac{dq(1-\beta^2)r\sin\theta}{4\pi\varepsilon_0((x-Vt)^2+(y^2+z^2)(1-\beta^2))^{3/2}},$$

where $dq = \tau dl$ is the charge value on the elementary segment dl; $\mathbf{r} = (x - Vt)\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is the radius vector drawn from the instantaneous position of charge $dq = \tau dl$ to the given field point A(x, y, z); $\tau = \tau'/\sqrt{1-\beta^2}$; $\beta = V/c$, θ is the angle between vector $\mathbf{r}$ and the direction of thread motion (figure 9).

Since we are looking for the electric field strength created at a given moment by the entire moving charged thread (and this means that at a given moment we need to integrate along the length of the thread in the reference frame K), the previous expression should be presented in the form

$$dE_z = \frac{d\cdot(1-\beta^2)\tau dl}{4\pi\varepsilon_0((x-l)^2+d^2(1-\beta^2))^{3/2}},$$

where l is the variable coordinate of the elementary segment dl.

We will integrate for the moment of time when the origins of the coordinate systems K and K' coincide. If we introduce a new variable $x - l = u$, then we get

$$E_z = \int\limits_{x-0}^{x-l'\sqrt{1-\beta^2}} dE_z = -\frac{d\cdot\tau(1-\beta^2)}{4\pi\varepsilon_0}\int\limits_{x-0}^{x-l'\sqrt{1-\beta^2}} \frac{du}{(u^2+d^2(1-\beta^2))^{3/2}} =$$

$$= -\frac{\tau\cdot d\cdot(1-\beta^2)}{4\pi\varepsilon_0}\left(\frac{u}{d^2(1-\beta^2)\sqrt{u^2+d^2(1-\beta^2)}}\right)\Bigg|_{x-0}^{x-l'\sqrt{1-\beta^2}}$$

Substitution of integration limits gives the value of the electric field strength of the sought electric field

$$E_z = \frac{\tau}{4\pi\varepsilon_0 d}\left(\frac{x}{\sqrt{x^2+d^2(1-\beta^2)}} - \frac{x-l'\sqrt{1-\beta^2}}{\sqrt{(x-l'\sqrt{1-\beta^2})^2+d^2(1-\beta^2)}}\right) \tag{21}$$

Let's find the magnetic field induction generated by a charged thread moving uniformly using the relativistic form of the Biot-Savart law (17)

$$dB = \frac{\mu_0}{4\pi}\frac{i[d\mathbf{l},\mathbf{r}](1-\beta^2)}{r^3(1-\beta^2\sin^2\theta)^{3/2}}\frac{1-\beta^2}{\sqrt{1-\beta^2}} = \frac{\mu_0}{4\pi}\frac{i[d\mathbf{l},\mathbf{r}](1-\beta^2)}{((x-l)^2+(y^2+z^2)(1-\beta^2))^{3/2}}$$

The y-component of the magnetic field induction at any point in the XOZ plane equals

$$dB_y = -\frac{\mu_0 idl(1-\beta^2)z}{4\pi((x-l)^2+d^2(1-\beta^2))^{3/2}} = \frac{\mu_0 i\cdot du(1-\beta^2)\cdot z}{4\pi(u^2+d^2(1-\beta^2))^{3/2}}, \tag{22}$$

where $x-l=u$, $du=-dl$, $i=\tau V=\tau' V/\sqrt{1-\beta^2}$.

Integrating (22) along the length of the moving charged thread, we get

$$B_y = \int\limits_{x-0}^{x-l'\sqrt{1-\beta^2}} dB_y = \frac{\mu_0 V\tau' d\cdot(1-\beta^2)}{4\pi\sqrt{1-\beta^2}} \int\limits_{x-0}^{x-l'\sqrt{1-\beta^2}} \frac{du}{(u^2+d^2(1-\beta^2))^{3/2}} =$$

$$= \frac{\mu_0 V\tau' d\cdot(1-\beta^2)}{4\pi\sqrt{1-\beta^2}} \left(\frac{u}{d^2(1-\beta^2)\sqrt{u^2+d^2(1-\beta^2)}}\right)\Bigg|_{x-0}^{x-l'\sqrt{1-\beta^2}}$$

Thus, we obtain the magnitude of the magnetic induction and electric field strength created by the moving charged filament at an arbitrary point in the XOZ plane:

$$B_y = \frac{\mu_0\tau' V}{4\pi d\cdot\sqrt{1-\beta^2}}\left[\frac{x-l'\sqrt{1-\beta^2}}{\sqrt{(x-l'\sqrt{1-\beta^2})^2+d^2(1-\beta^2)}} - \frac{x}{\sqrt{(x)^2+d^2(1-\beta^2)}}\right] =$$

$$= -\frac{\mu_0\tau' V}{4\pi d\cdot\sqrt{1-\beta^2}}\left[\frac{l'\sqrt{1-\beta^2}-x}{\sqrt{(x-l'\sqrt{1-\beta^2})^2+d^2(1-\beta^2)}} + \frac{x}{\sqrt{(x)^2+d^2(1-\beta^2)}}\right]$$

$$E_z = \frac{\tau}{4\pi\varepsilon_0 d}\left[\frac{x}{\sqrt{x^2+d^2(1-\beta^2)}} + \frac{l'\sqrt{1-\beta^2}-x}{\sqrt{(x-l'\sqrt{1-\beta^2})^2+d^2(1-\beta^2)}}\right]$$

If the origin of coordinates of reference frame K is positioned so that the OZ axis passes through the field point A, then the integration limits will be from $-x$ to $l'\sqrt{1-\beta^2}-x$, and integration along the length of the moving filament gives the same result:

$$E_z = \int\limits_{-x}^{l'\sqrt{1-\beta^2}-x} dE_z = \frac{\tau'}{4\pi\varepsilon_0 d\sqrt{1-\beta^2}}\left[\frac{l'\sqrt{1-\beta^2}-x}{\sqrt{\left(l'\sqrt{1-\beta^2}-x\right)^2+d^2(1-\beta^2)}} + \frac{x}{\sqrt{x^2+d^2(1-\beta^2)}}\right].$$

$$B_y = -\frac{V\cdot\tau'\mu_0}{4\pi\cdot d\sqrt{1-\beta^2}}\left[\frac{l'\sqrt{1-\beta^2}-x}{\sqrt{\left(l'\sqrt{1-\beta^2}-x\right)^2+d^2(1-\beta^2)}} + \frac{x}{\sqrt{x^2+d^2(1-\beta^2)}}\right].$$

The values of the electric field strength and magnetic induction E', B', E, B in all formulas of this problem are taken for the same space-time point. If we take the moment when the origins of the coordinate systems K and K' coincide ($t=t'=0$, then $x'=x/\sqrt{1-\beta^2}$) and from the last two formulas we obtain

$$E_z = \frac{E_z'}{\sqrt{1-\beta^2}}; \quad B_y = -\frac{VE_z'}{c^2\sqrt{1-\beta^2}} \tag{23}$$

However, if we had used the Biot-Savart law in its classical form to find B_y, we would have obtained a result that contradicts (*) and (23).

This means that the application of the classical Biot-Savart law (18) to find the electromagnetic field of a uniformly and translationally moving charged filament gives results that contradict the requirements of relativistic electrodynamics.

On the other hand, when finding the total force of interaction between two charged moving parallel filaments (appendix A) as a result of using formulas (5) and (17), we obtain a result that does not contradict the requirements of special relativity: $F_z = F^E{}_z = F^B_z = F'_z \cdot \sqrt{1-\beta^2}$.

Obviously, if we found the force of interaction between the filaments based on the classical Biot-Savart law, this requirement would not be met.

Thus, the examples considered above show that the Biot-Savart law in its relativistic form gives a correct description of physical phenomena, while the application of the classical Biot-Savart law (18) does not give a relativistically invariant description of these phenomena.

Therefore, its application in the process of teaching classical electrodynamics does not allow for a complete and exhaustive analysis of the phenomena that arise during the rectilinear and uniform motion of charge carriers.

6. The mechanism of magnetic field generation by quasi-stationary currents and equivalent descriptions of phenomena in electrodynamics

Within the framework of the proposed methodology for forming the concept of a magnetic field, we have obtained a set of important scientific and methodological results (sections 4, 5).

Thus, the results obtained in section 5 are a consequence of the requirements of special relativity regarding the description of electromagnetic interaction, but special relativity cannot provide or explain the mechanism of the emergence of the magnetic field of currents. *In particular, it only indicates what the relationships between quantities should be for the description and analysis of electrodynamic phenomena to be correct.*

Let us once again note the main provisions of the “traditional”, orthodox methodology for describing the properties of the magnetic field of stationary and quasi-stationary currents.

As already noted (see [31, 43, 49, 53]), Maxwell’s equation

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \tag{24}$$

means that the vortex magnetic field is generated by conduction currents. Moreover, at points in space where there is a current density $\mathbf{j}(\mathbf{r})$, the curl of the magnetic induction vector is equal to $\mu_0\mathbf{j}$. That is, at points in space where there are no conduction currents, $\nabla \times \mathbf{B} = 0$, the magnetic field is not zero, despite the fact that the current density, as the source of the magnetic field, is zero at these points.

Let us also note the following methodological difficulty: how to physically explain the emergence of a magnetic field at points in space where there are no conduction currents; how can it be that conduction currents are not zero in one part of space, and a magnetic field appears at those points in space where there are no currents? So what and how creates the magnetic field at these points?!

Answers to some of these questions can be provided by the methodological approach first proposed by Nikolaev [51], then developed by us in works [18, 24, 25] and finally implemented in textbooks [15, 16, 30, 31] in the case of non-relativistic motion of charged particles.

Thus, in Irodov's textbooks [15, 16], the expression for the magnetic induction of a moving charged particle at $v \ll c$ is obtained as a result of applying Maxwell's integral equation for displacement currents.

Moreover, the mechanism or cause of the emergence of a magnetic field due to the motion of a charged particle is not analyzed here at all. It is simply shown as a result of calculations [15, 16] that the circulation of the magnetic induction vector, which is created by a uniformly moving charged particle, along a circular contour that symmetrically encompasses the line of motion of the charged particle, is related to the displacement current through the surface resting on this contour.

On the other hand, in our proposed approach [18, 21, 24, 25, 30, 31] *for non-relativistic motion of charged particles*, for the first time in the scientific and methodological literature, the idea of the generation of a magnetic field of moving charged particles by displacement currents appears.

And in works [18, 21, 25], we show for the first time that the physical reason for the emergence of a magnetic field due to the motion of charged particles (*moreover, at an arbitrary magnitude of the velocity of motion of the charged particle*) is the displacement current density $\mathbf{j}_{\text{disp}} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$.

In section 5, it is shown that from the requirements of the principle of relativity and Coulomb's law, the electric field strength and magnetic induction under the condition of uniform motion of a charged particle with an arbitrary magnitude of velocity are determined by (5) and (12). But, as already noted, regarding the physical mechanism of the emergence of a magnetic field during the motion of charged particles, special relativity can say nothing.

Based on the properties of the electromagnetic field of a uniformly moving charged particle (5) and (12), we can make sure that between the quantities characterizing the electromagnetic field of a moving charged particle in a homogeneous and isotropic medium, there are connections

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \tag{25}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{26}$$

Note that the fundamental equation (25) should be considered as a consequence of the properties of the electromagnetic field of a charged particle moving uniformly and rectilinearly (or, in other words, as a consequence of the principle of relativity and Coulomb's law).

We have shown that the magnetic field of a proton at any point in space outside the moving charge, which moves with an arbitrary constant velocity, is generated only by displacement currents. For non-relativistic motion of charged particles, a partial discussion of a similar issue is given in [15, 16, 51] and in the author's textbook [31].

Indeed, let's consider ***Example 1.2.*** A proton moves along the OX axis with a constant velocity $v < c$ (figire 10). Using (25), find the magnetic field at an arbitsrary point in space outside the charged particle.

Solution: The magnetic field at point $\mathrm{P}(x, y, z)$ cannot be generated by transfer currents (conduction currents), since the transfer current through the flat surface of a circle of radius ρ_0, on the periphery of which point P is located, is zero ($i_{\text{trans}} = \int\limits_S \mathbf{j} \cdot d\mathbf{S} = 0$, because $\mathbf{j} = 0$).

Let's use (25) to find the magnetic field strength of a moving charged particle based on the known electric field strength (5).

According to (25), the appearance of a magnetic field at point $\mathrm{P}(x, y, z)$ is caused by displacement currents.

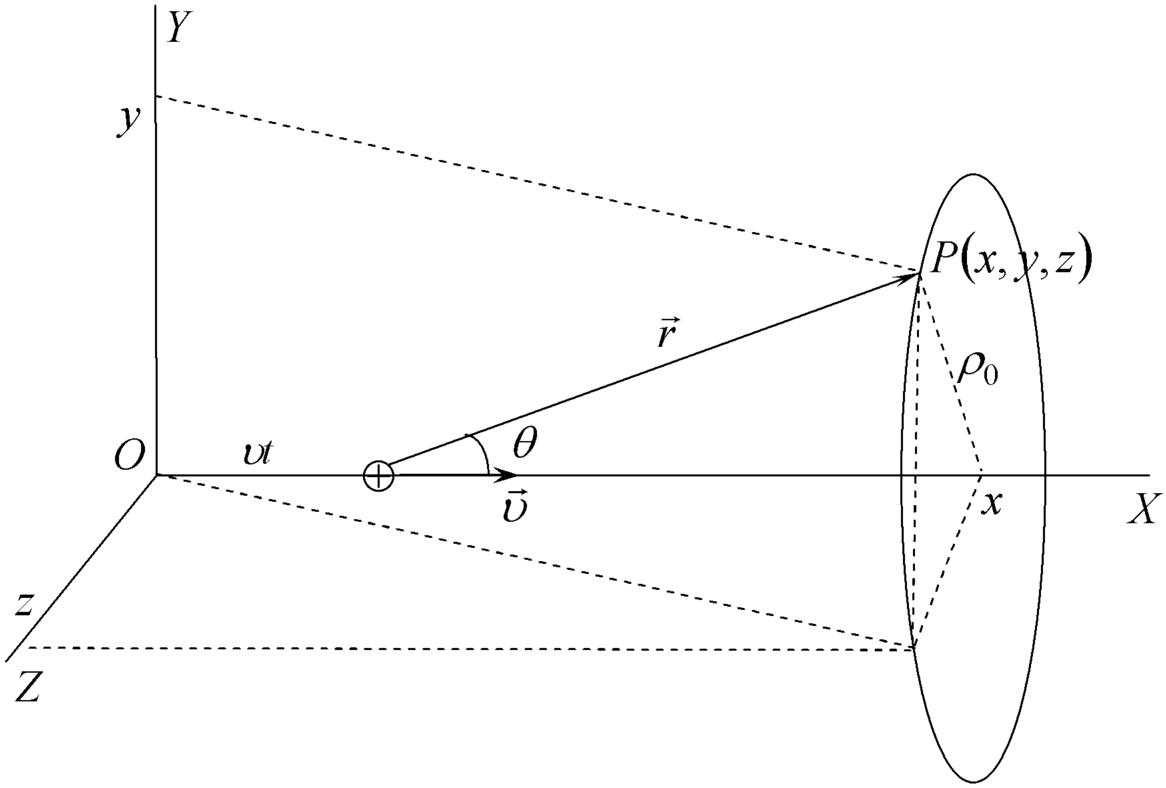

Figure 10: Displacement current and magnetic field of a proton moving at speed $\mathbf{v} = const$.

Indeed, since at an arbitrary point in space (point $P(x, y, z)$) the electric field strength created by the motion of a charged particle ($\mathbf{v}$ = const) is determined by (5), then for the displacement current density at this point in space we have

$$\mathbf{j}_{\text{disp}} = \frac{\partial \mathbf{D}}{\partial t} = \left[\frac{3qv(1-\beta^2)(x-vt)^2}{4\pi\left[(x-vt)^2+\rho_0^2(1-\beta^2)\right]^{3/2}} - \frac{qv(1-\beta^2)}{4\pi\left[(x-vt)^2+\rho_0^2(1-\beta^2)\right]^{3/2}}\right]\mathbf{i} + \\ + \frac{3qvy(1-\beta^2)(x-vt)}{4\pi\left[(x-vt)^2+\rho_0^2(1-\beta^2)\right]^{5/2}}\mathbf{j} + \frac{3qvz(1-\beta^2)(x-vt)}{4\pi\left[(x-vt)^2+\rho_0^2(1-\beta^2)\right]^{5/2}}\mathbf{k} \tag{27}$$

For the displacement current i_{disp} through the flat surface S_0 we obtain

$$i_{disp} = \int\limits_{S_0} \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S} = \frac{1}{2}\int\limits_0^{\rho_0} \left[\frac{3qv(1-\beta^2)(x-vt)^2}{\left[(x-vt)^2+\rho_0^2(1-\beta^2)\right]^{5/2}} - \frac{qv(1-\beta^2)}{\left[(x-vt)^2+\rho_0^2(1-\beta^2)\right]^{3/2}}\right]\rho d\rho = \\ = \frac{qv(1-\beta^2)\rho_0^2}{2\left[(x-vt)^2+\rho_0^2(1-\beta^2)\right]^{3/2}}$$

When applying the equation $\oint\limits_L \mathbf{H} \cdot d\mathbf{l} = \int\limits_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$, the theorem on the circulation of the vector $\mathbf{H}$ takes the form

$$2\pi\rho_0 H = \frac{qv(1-\beta^2)\rho_0^2}{2\left[(x-vt)^2+\rho_0^2(1-\beta^2)\right]^{3/2}}$$

From this

$$H = \frac{qv(1-\beta^2)\rho_0}{4\pi\left[(x-vt)^2+\rho_0^2(1-\beta^2)\right]^{3/2}}$$

which exactly corresponds to formula (12) and is confirmed by experimental facts.

Given the local nature of the basic laws of classical and relativistic electrodynamics, let's now find the magnetic field strength at point $P(x, y, z)$ using Maxwell's equation in differential form (25). Note that according to [25] $H_x = 0$, so the curl of the vector $\mathbf{H}$ is equal to

$$\nabla \times \mathbf{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{i} + \frac{\partial H_z}{\partial x}\mathbf{j} - \frac{\partial H_y}{\partial x}\mathbf{k}$$

Then, taking into account (27), we obtain a system of partial differential equations for the unknowns $H_z(x, y, z)$ and $H_y(x, y, z)$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \frac{3qv(1-\beta^2)(x-vt)^2}{4\pi[(x-vt)^2+\rho_0^2(1-\beta^2)]^{5/2}} - \frac{qv(1-\beta^2)}{4\pi[(x-vt)^2+\rho_0^2(1-\beta^2)]^{3/2}} \tag{28}$$

$$\frac{\partial H_z}{\partial x} = -\frac{3qv(1-\beta^2)(x-vt)y}{4\pi[(x-vt)^2+\rho_0^2(1-\beta^2)]^{5/2}}$$

$$\frac{\partial H_y}{\partial x} = \frac{3qv(1-\beta^2)(x-vt)z}{4\pi[(x-vt)^2+\rho_0^2(1-\beta^2)]^{3/2}}$$

Integrating the last two equations, we find $H_z(x, y, z)$ and $H_y(x, y, z)$

$$H_z(x, y, z, t) = \frac{qv(1-\beta^2)y}{4\pi[(x-vt)^2+\rho_0^2(1-\beta^2)]^{3/2}} \tag{29}$$

$$H_y(x, y, z, t) = -\frac{qv(1-\beta^2)z}{4\pi[(x-vt)^2+\rho_0^2(1-\beta^2)]^{3/2}} \tag{30}$$

By direct verification, one can make sure that the last two solutions satisfy equation (28).

The transfer current through the surface S_0, assuming that the substance responsible for the properties of the charge is uniformly distributed over the surface of the charged particle of radius r_0, is equal to

$$i_{\text{trans}} = \frac{dq}{dt} = \frac{d}{dt}(\sigma 2\pi r_0 dx) = \frac{qv}{2r_0}$$

This transfer current will create a magnetic field only at points in space limited by the dimensions of the charged particle. And outside the charged particle, the magnetic field is created only by displacement currents.

In any case, this is true for uniform and rectilinear motion of a proton.

According to the modern physical paradigm, any interaction propagates with a finite speed, from one point in space to another infinitely close point. This means that the laws of physics, and in particular electrodynamics, must be formulated in local form.

In our case, this means that the *cause of the magnetic field* $\mathbf{H}$ *at point* $P(x, y, z)$ *is a phenomenon that occurs at this same point or in its vicinity.*

Given the field concept of interaction, the cause of the magnetic field at some point in space associated with the motion of charged particles should be recognized as the change in time of the vector $\mathbf{D}$ ($\frac{\partial \mathbf{D}}{\partial t}$) at this same point in space.

The Biot-Savart law, the theorem on the circulation of the magnetic field strength vector, and other integral laws with which the magnetic fields of constant currents are determined do not explain the cause of the magnetic field at points in space where there are neither charges nor transfer current.

Since in modern scientific and methodological literature on electrodynamics there is no physical justification for the mechanism of the magnetic field during the motion of charged particles, the above explanations of this phenomenon are methodologically and *methodologically relevant and important in the context of professional training of physics teachers.*

Let's consider a more general problem and in a practical sense more relevant. Namely: let's show that the magnetic field of a linear conductor with current (of finite or infinite length) at any point in space outside the current-carrying conductor is generated only by displacement currents.

Example 1.3. For this, let's consider a conductor of length l_0 with a current caused by the motion of charged particles with a constant drift velocity (figure 11). The solution to this problem was first presented by us in [18].

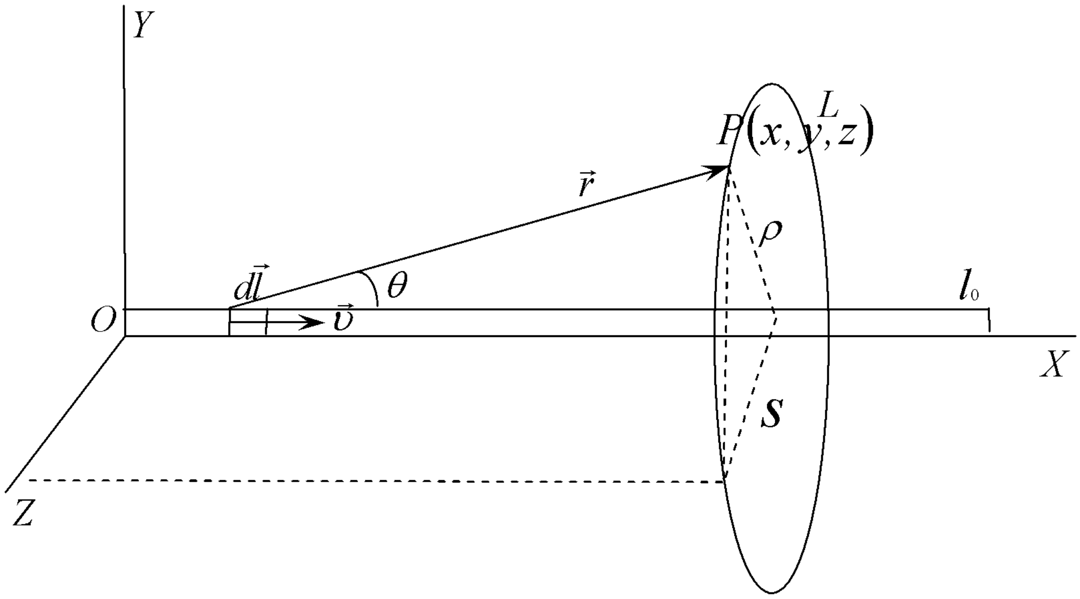

Figure 11: The explanation for determining the displacement current density at point P of a direct current segment.

Solution: Each elementary segment of a linear conductor with current can be considered as a charge of magnitude $dq = \tau dl$, moving with $\mathbf{v} = \text{const}$ (where $\tau = \frac{qnS}{\sqrt{1-\beta^2}}$ is the linear charge density, q is the magnitude of the charge of each of the particles which form the current due to motion, n is their concentration, S is the cross-sectional area of the conductor).

Then, the total displacement current density at an arbitrary point P(x, y, z) is equal to

$$\mathbf{J}_{\text{disp}} = \int_0^{l_0} \mathbf{j}_{\text{disp}} \cdot dl = \left[-\frac{u\tau \cdot u^3}{4\pi\rho^2[u^2+\rho^2(1-\beta^2)]^{3/2}} + \frac{u\tau \cdot u}{4\pi\rho^2[u^2+\rho^2(1-\beta^2)]^{1/2}} \right] \Bigg|_0^{l_0} \cdot \mathbf{i} + \left[\frac{v\tau \cdot (1-\beta^2)}{4\pi[u^2+\rho^2(1-\beta^2)]^{3/2}} \cdot (y\mathbf{j} + z\mathbf{k}) \right] \Bigg|_0^{l_0} \tag{31}$$

where $u = x - l_0$.

Taking, for example, only the $(\mathbf{J}_{\text{disp}})_y$ component and using (24) to find H_z after integrating the corresponding equation, we obtain

$$H_z(y) = \frac{\tau v y}{4\pi\rho^2} \left[\frac{l_0 - x}{\sqrt{(x-l_0)^2+\rho^2(1-\beta^2)}} + \frac{x}{\sqrt{x^2+\rho^2(1-\beta^2)}} \right] \tag{32}$$

For the magnetic field strength at point P(x, y, z), since $H = H_z \cdot \frac{y}{\rho}$, which arises due to the motion of charge carriers along a conductor segment of length l_0, we obtain the expression

$$H = \frac{\tau v}{4\pi\rho}\left[\frac{l_0 - x}{\sqrt{(x-l_0)^2+\rho^2(1-\beta^2)}} + \frac{x}{\sqrt{x^2+\rho^2(1-\beta^2)}}\right] \tag{33}$$

which in the non-relativistic case coincides with the well-known formula for the magnitude of the vector $\mathbf{H}$ of a conductor segment of length l_0 with current $i = v\tau$ (see also formula (20)).

The displacement current through the flat surface of a circle of radius ρ

$$I'_{\text{disp}} = \int_0^{l_0} dx \int_0^{\rho} \mathbf{j}_{\text{disp}} \cdot 2\pi\rho d\rho \cdot \mathbf{i} = \frac{\tau v}{2}\left[\frac{l_0 - x}{\sqrt{(x-l_0)^2+\rho^2(1-\beta^2)}} + \frac{x}{\sqrt{x^2+\rho^2(1-\beta^2)}}\right] \tag{34}$$

Only in the case when the conductor is infinitely long, $I_{\text{disp}} = v\tau$ (for any radius ρ) and is equal to I_{trans} – the so-called transfer current.

And the displacement current through the entire surface $S - \pi\rho^2$, which borders with $\pi\rho^2$ is equal to:

$$I''_{\text{disp}} = -\frac{\tau v}{2}\left[\frac{l_0 - x}{\sqrt{(x-l_0)^2+\rho^2(1-\beta^2)}} + \frac{x}{\sqrt{x^2+\rho^2(1-\beta^2)}}\right] \tag{35}$$

Thus, $I'_{\text{disp}} = -I''_{\text{disp}}$, the displacement current I'_{disp} through the surface $S = \pi\rho^2$ is closed by the current I''_{disp} through the surface $S - \pi\rho^2$ that borders with $\pi\rho^2$.

This conclusion is valid for a flat surface of any radius and at any velocity of charge carriers.

Analysis of these problems in the non-relativistic case was carried out in the textbook [31, p. 86-92], the implementation of which in the educational process has been taking place over the past 10 years.

For velocities $v \ll c$, this property of displacement currents was first noted in [51]. But unlike the situation shown in figure 2 in [24], in which the transfer current and displacement current through S_0 are added, when considering a conductor with current (figure 11), the real (physical) current through the surface $S = \pi\rho^2$ is equal only to the displacement current (1.34) (see also [51], [31, p. 86-92]).

Thus, it has been shown that the *physical cause of the magnetic field of stationary and quasi-stationary currents is only the displacement current.*

The situation when one and the same subject area of physics or some of its parts are described by two or more theories, or fragments of theories, and which lead to the same empirical consequences, is called "equivalent descriptions" [4, p. 42]. In principle, the conclusion about the existence of equivalent descriptions follows, generally speaking, from Einstein's epistemological positions. He argued that the same aspect of physical reality can be described by different theoretical constructions: "Corresponding to the same complex of empirical data, there may be several theories, which differ from one another to a considerable extent. But as regards the deductions from the theories which are capable of being tested, the agreement between the theories may be so complete that it becomes difficult to find any deductions in which the two theories differ from each other." [6, p. 106].

At the same time, he often emphasized that the fundamental concepts and laws of physical theory are "free creations of the human mind" [12, p. 43]. "There is no inductive method which could lead to the fundamental concepts of physics" [8, p. 365]. "knowledge cannot spring from experience alone but only from the comparison of the inventions of the intellect with observed fact" [11, p. 266] – only theory can say what is measured in an experiment: "Physics constitutes a logical system of thought which is in a state of evolution, whose basis cannot be distilled, as it were, from experience by an inductive method, but can only be arrived at by free invention" [10, p. 78].

That is, the fundamental irreducibility of theory to physical experiments means that the same area of facts can be described by several theoretical models.

An example of such an equivalent description is the three formulations of non-relativistic quantum mechanics – the Schrödinger representation, the Heisenberg representation, and the formulation of quantum mechanics in the language of path integrals (R. Feynman) [4, 37]. A second example of equivalent description is the formulation of special relativity – in Lorentz and Galilean coordinates [4, 44].

As a rule, although equivalent descriptions lead to the same empirical consequences, they sometimes give different, and even incompatible, ideas about the physics of phenomena and the objective picture of the world.

But if a certain complex of experimental facts can be described by different but equivalent theories, it is difficult to answer the question of which of them corresponds to reality and what is true.

This problem can be formulated more narrowly, speaking only about different descriptions of a number of electrodynamic problems. Therefore, from simpler examples of equivalent descriptions of electrodynamic processes, we give the following:

A. The problem of capacitor discharge.

In the traditional methodology, it is described by Kirchhoff's second law, but without special physical explanations. However, from the point of view of field representations, this discharge process is caused by the migration of field energy from the volume of the capacitor into the external space, and then by its inflow into the conductor that short-circuits the capacitor plates. In this case, one cannot do without Maxwell's equation, ideas about displacement currents, and the Poynting vector. Although the final result is obtained the same, obviously, the physical interpretation is completely different.

B. The problem of the physical mechanism of alternating current flow through a capacitor (figure 12).

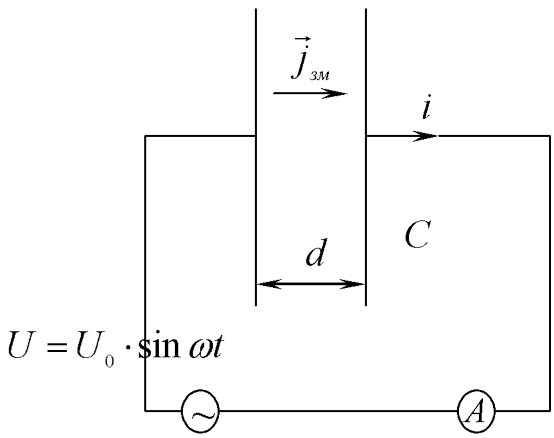

Figure 12: The phenomenon of alternating current flow through a capacitor.

Two equivalent descriptions of this process can be indicated. The first is based on the solution of Kirchhoff's second rule, the second is based on the idea that the current flowing through the capacitor is completely caused by displacement currents, and the ammeter measures exactly the displacement current.

Unlike the first, the second method gives an adequate physical explanation of the mechanism of this process (for more details, see the author's manual [31, p. 66-70]).

C. The problem of calculating the amount of heat released in a conductor with active resistance $R, -i^2R$.

At the electrical engineering level of teaching and explaining this phenomenon, this amount of heat is caused by the collision of conduction electrons with ions of the crystal lattice and the transfer of their kinetic energy to these ions. From the point of view of adequate physical concepts, this amount of heat is equal to the energy of the electromagnetic field that flows into the conductor from the external space.

Again, despite the same final result, the physical explanations are fundamentally different. Moreover, as experience in teaching at a pedagogical university shows, the electrical engineering level of explanation of these phenomena significantly distorts students' understanding of these (and other) electrodynamic processes. The idea that the energy carrier of the current is the electromagnetic field, which is localized both in the conductor and in the surrounding space, and not the current carriers (charged particles) is perceived by students with certain difficulties.

- **D.** The problem of calculating the magnetic field induction of a uniformly moving charged particle.

Its solution can be obtained in the following ways:

1. Using the long-range Biot-Savart law, which is valid at the velocity of current carriers $v \ll c$ [16, 31].
2. Based on Maxwell's equation $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j} + qv\delta(\mathbf{r} - \mathbf{r}'(t))$, where $\mathbf{v}$ is the velocity of the charged particle, q is the charge magnitude, $\delta(\mathbf{r} - \mathbf{r}(t))$ is the Dirac delta function, $\mathbf{r}$ is the radius vector drawn from the instantaneous position of the charged particle to the given field point [30, 43].
3. Using Liénard–Wiechert potentials [2, 30, 31];
4. Based only on the concept of displacement currents and equation (25) [21, 25];
5. Based on Coulomb's law and the principle of relativity [33].

Let's focus in more detail on method **D**4, in order to compare it later with the description of this problem based on method **D**1.

In works [21, 33] (see also section 6) *based on the concept of displacement currents*, it was found that the magnetic field strength of a uniformly moving charged particle (at any magnitude of its motion velocity) is equal to

$$\mathbf{H}(x, y, z, t) = \frac{q\mathbf{v} \times \mathbf{r}}{4\pi} \cdot \frac{1 - \beta^2}{[(x - vt)^2 + (y^2 + z^2)(1 - \beta^2)]^{3/2}}$$

Therefore, to find the magnetic field induction at an arbitrary point in space, which is created by a "current element", we can obviously write

$$d\mathbf{B} = \mu_0 \mathbf{H} \cdot N = \frac{\mu_0 i}{4\pi} \cdot \frac{[d\mathbf{l} \times \mathbf{r}](1 - \beta^2)}{r^3(1 - \beta^2 \sin^2\theta)^{3/2}} \tag{36}$$

where N is the number of charged particles that realize the "current element".

Thus, with the help of obvious algebraic transformations, we managed to transform one description into another description.

That is, at each point in space at some point in time, the curl of the vector $\mathbf{H}$ is caused (generated) by the displacement current density at this same point and at this same moment in time.

Generalizing equations (25) and (26) to electromagnetic fields created by uniform ordered motions of charged particles, we come to the following conclusion.

Equations (25) and (26) not only explain the physical mechanism of generation of field vectors $\mathbf{D}$ and $\mathbf{B}$ of the electromagnetic field associated with uniform displacements of charged particles in space, but also allow finding the values of these vectors based on known derivatives $\frac{\partial \mathbf{B}}{\partial t}$ and $\frac{\partial \mathbf{D}}{\partial t}$.

E. The problem of finding the magnetic field induction created by a linear conductor with constant current (of finite or infinite length) at an arbitrary point in space outside the conductor with direct current is found, as is known,

1) using the Biot-Savart law and the superposition principle;
2) by solving Laplace's equation $\nabla^2 \mathbf{A} = 0$, where $\mathbf{B} = \nabla \times \mathbf{A}$, $\mathbf{A}(x, y, z)$ is the vector potential [30, 31];
3) using equation (25), as we have shown in [24, 31, 33].

Let's compare the method of solving this problem **E**.3 with methods **E**.1 and **E**.2. Since the solution by methods **E**.1 and **E**.2 is described in standard electrodynamics textbooks, there's no need to dwell on it.

The description of the previous problem showed that when a charged particle moves, a magnetic field is created at each point in space, which (according to the principle of local action) is generated by an electric field variable in time (i.e., by displacement currents) at this same point in space. *Therefore, based on the superposition principle, the magnetic field of a conductor with constant current should be generated only by displacement currents.*

That is, based on the concept of displacement currents, and assuming that at an arbitrary point in space outside the conductor with direct current $\nabla \times \mathbf{H} \neq 0$ (Example 1.2), we obtained the same result (33) as with the traditional description of this problem. But the traditional description, in principle, is not able to explain the mechanism and nature of the magnetic field, it provides results that are confirmed by experiments and experience, in the language of quantities that are experimentally directly measurable: current strengths, voltages, active resistances.

And in the minds of students, the impression is formed that physics deals mainly with phenomenological problems and is a purely experimental-research science.

However, in this case (as in others considered above) we again see that only theory can say what is measured or observed in an experiment.

"The prejudice–which has by no means died out in the meantime–consists in the faith that facts by themselves can and should yield scientific knowledge without free conceptual construction. Such a misconception is possible only because one does not easily become aware of the free choice of such concepts, which, through verification and long usage, appear to be immediately connected with the empirical material." [9, p. 13].

As we can see, the physical reason for the emergence of a magnetic field in the vicinity of a conductor with direct current is only the displacement current, and it is the students' awareness of this fact that can ensure the formation of a correct scientific picture of the world in them. Of course, one can say that each of the descriptions within the framework of a separate problem complements the other. But it is natural to ask the question: which of the descriptions corresponds to reality.

Therefore, it is necessary to create such a methodology for teaching electrodynamics that would ensure awareness of the deep differences between equivalent descriptions in general and between equivalent descriptions based on fundamental non-empirical approaches in the representation of objective physical reality. Quite aptly, Chudinov

[4, p. 47] noted on this matter: “The change in the language of scientific theory, unlike the translation of text from one natural language to another, is reflected in the vision of essential aspects of the structure of the objective world”.

And the examples considered above, in our opinion, provide the implementation of such an approach.

7. The principle of relativity and the phenomenon of electromagnetic induction

As a result of discussions in educational and methodological literature [13, 16, 47] regarding the physical causes of induced EMF, the accepted view is that there is a dual nature to the induced EMF. If a constant but non-uniform magnetic field exists in the laboratory reference frame K, then the EMF induced in a contour moving in this field is caused by the Lorentz force. In the contour’s own reference frame K’, the EMF is caused by the emergence of an electric field generated by the motion of the magnetic field from K relative to K’. An observer in K’ can also interpret “their” EMF as a consequence of the change in magnetic induction over time.

In fact, in Einstein’s first paper on special relativity “On the Electrodynamics of Moving Bodies” [5], this is emphasised: “It is known that Maxwell’s electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion” [5, p. 891].

As analysis shows (see sections 3.1, 3.2 of the author’s monograph [33] and [16, 31]), in most cases the EMF is determined by the rate of change of magnetic flux through the contour and does not depend on how the change in magnetic flux Φ is realised (“flux rule”):

$$\varepsilon = \frac{d\Phi}{dt} \tag{37}$$

“We know of no other place in physics where such a simple and accurate general principle requires for its real understanding an analysis in terms of two different phenomena. Usually such a beautiful generalization is found to stem from a single deep underlying principle. Nevertheless, in this case there does not appear to be any such profound implication. We have to understand the “rule” as the combined effects of two quite separate phenomena.” (R. Feynman [13, chap. 17])

However, when analysing specific physical situations in which electromagnetic induction is observed, one should always pay attention to the physical cause of the EMF.

Irodov [16] repeats Feynman’s words regarding the absence of a unifying principle underlying the law of electromagnetic induction: “Given that no single deep principle unifying both phenomena is apparent, we must perceive the law of electromagnetic induction as the combined effect of two completely different phenomena. Both these phenomena are generally independent of each other, and yet – surprisingly – the induced EMF in a circuit is always equal to the change in magnetic flux through the circuit” [16, p. 230].

In general physics courses, when studying electromagnetic induction, attention is usually focused on two physical causes of induced EMF in a closed contour or in its individual parts [13, 16, 47, 56]: the action of the Lorentz force on free electrons in a conductor moving in a magnetic field, and the emergence of a vortex electric field in a non-stationary magnetic field.

Standard methods for teaching physics in secondary schools propose studying the patterns of this phenomenon in two stages [1, 4, 24, 45]. First, electromagnetic induction in moving conductors is analysed using the traditional model (figure 13), and then, based on known experiments illustrating this phenomenon, students form ideas about the vortex electric field [59].

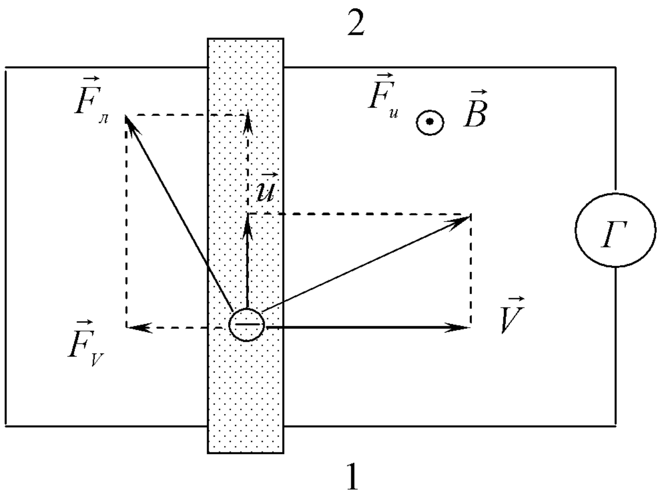

Figure 13: A traditional model used for justification "flux rule" (37).

Analysis of scientific and methodological publications on the interpretation of electromagnetic induction and methods for studying it reveals a number of methodological problems [1, 14, 32] (see also section 3.1 of the monograph [33]).

Indeed, when justifying the local form of the electromagnetic induction law in several electrodynamics textbooks ([56, p. 233], [39, p. 350], [53, p. 107]), the transition from total derivative to partial derivative, and vice versa, remains unclear and unjustified. The derivation of the equation $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ from the "flux rule" (37) is incorrect and inconsistent.

The conclusion in the physics encyclopaedia [55] that the relations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

or

$$\oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{S} = -\frac{\partial \Phi}{\partial t}$$

are universally applicable [55, p. 537] to arbitrary moving media and systems contradicts the corresponding provisions of D. V. Sivukhin's textbook [60, p. 271-272] and the essence of these Maxwell equations.

That is, this conclusion is erroneous, since these Maxwell equations describe electromagnetic induction in stationary media and contours.

While Gauss's electrostatic theorem, the theorem on the circulation of the vector B, and other Maxwell equations in integral form correspond to Maxwell equations in differential form, for the "flux rule" (37), as seen from the analysis of scientific and methodological literature [33], no corresponding local law has been formulated.

One aspect of the modern physical paradigm assumes that an integral law is a consequence of a local law. That is, based on the essence of electrodynamics as a local and field theory, the integral form of the electromagnetic induction law should be a consequence of the local form of this law. The basic law should be considered the definition of $\nabla \times \mathbf{E}$ at an arbitrarily moving point in space, not just at a stationary point ($\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$). However, regarding the Maxwell equation $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, a completely different approach is adopted in the educational and methodological literature (see section 3.1 of the monograph [33]).

Further, the established assertion that the law of electromagnetic induction in integral form ("flux rule") (37) is an experimental law is considered erroneous.

Understanding that law (37) is not an experimental law is extremely important both scientifically and methodologically. This conclusion is important for understanding the relationship between theoretical and empirical aspects in the process of cognition of physical phenomena and the formulation of laws and principles describing these phenomena.

In the educational and methodological literature, the "flux rule" (37) is derived from the analysis of figure 13 and based on the expression for the Lorentz force as an external force. Due to its mathematical simplicity and transparency, this method of justifying (37) is the most common in the scientific and methodological literature.

At the same time, despite the fact that electrodynamics essentially belongs to relativistic physics, the analysis of a significant part of educational models of electrodynamics is carried out within the framework of classical mechanics.

Thus, when considering the traditional model (figure 13, [1, p. 24] [48, p. 12]), the velocity addition formulas, as well as the formulas for transforming the components of the electromagnetic field, are taken only in the non-relativistic approximation [1, 24, 45, 56]. The latter does not contribute to a deep understanding of the essence of the phenomenon and the principle of relativity, and sometimes leads to factual errors.

Thus, it can be said that in describing the phenomenon of electromagnetic induction, a number of provisions (both physical and methodological) are not sufficiently clearly formulated and are not consistent. All this requires a more careful and in-depth analysis of the phenomenon of electromagnetic induction and methods for studying it.

Let us show that the phenomenon of electromagnetic induction and the law of electromagnetic induction are consequences of the principle of relativity and Coulomb's law.

For the vector $\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E}$ of the electric field of a uniformly moving charged particle [29, 32, 33, 36] we have:

$$\mathbf{D} = \frac{q}{4\pi} \frac{1-\beta^2}{(1-\beta^2 \sin^2\theta)^{3/2}} \frac{\mathbf{r}}{r^3} = \frac{q(1-\beta^2)}{4\pi[(x-vt)^2 + (y^2+z^2)(1-\beta^2)]^{3/2}} \mathbf{r} \tag{38}$$

In the reference frame K associated with the contour, the charged particle moves with some velocity. But the electric field of a moving charged particle is *non-potential*, and this is of fundamental importance (see below).

Indeed, using (38), for the rotor of the vector $\mathbf{D}$ we find:

$$\begin{aligned} \nabla \times \mathbf{D} &= \left(\frac{\partial D_z}{\partial y} - \frac{\partial D_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial D_x}{\partial z} - \frac{\partial D_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial D_y}{\partial x} - \frac{\partial D_x}{\partial y}\right)\mathbf{k} \\ &= \frac{3q\beta^2(1-\beta^2)}{4\pi[(x-vt)^2 + (y^2+z^2)(1-\beta^2)]^{5/2}} [(x-vt)z\mathbf{j} - (x-vt)y\mathbf{k}] \end{aligned} \tag{39}$$

To simplify expressions and for greater transparency of calculations and conclusions, we will consider the electromagnetic field in the XOY plane (figure 15). Then

$$\nabla \times \mathbf{D} = -\frac{3q\beta^2(1-\beta^2)}{4\pi\varepsilon_0[(x-vt)^2 + y^2]^{5/2}} (x-vt)y\mathbf{k} \tag{40}$$

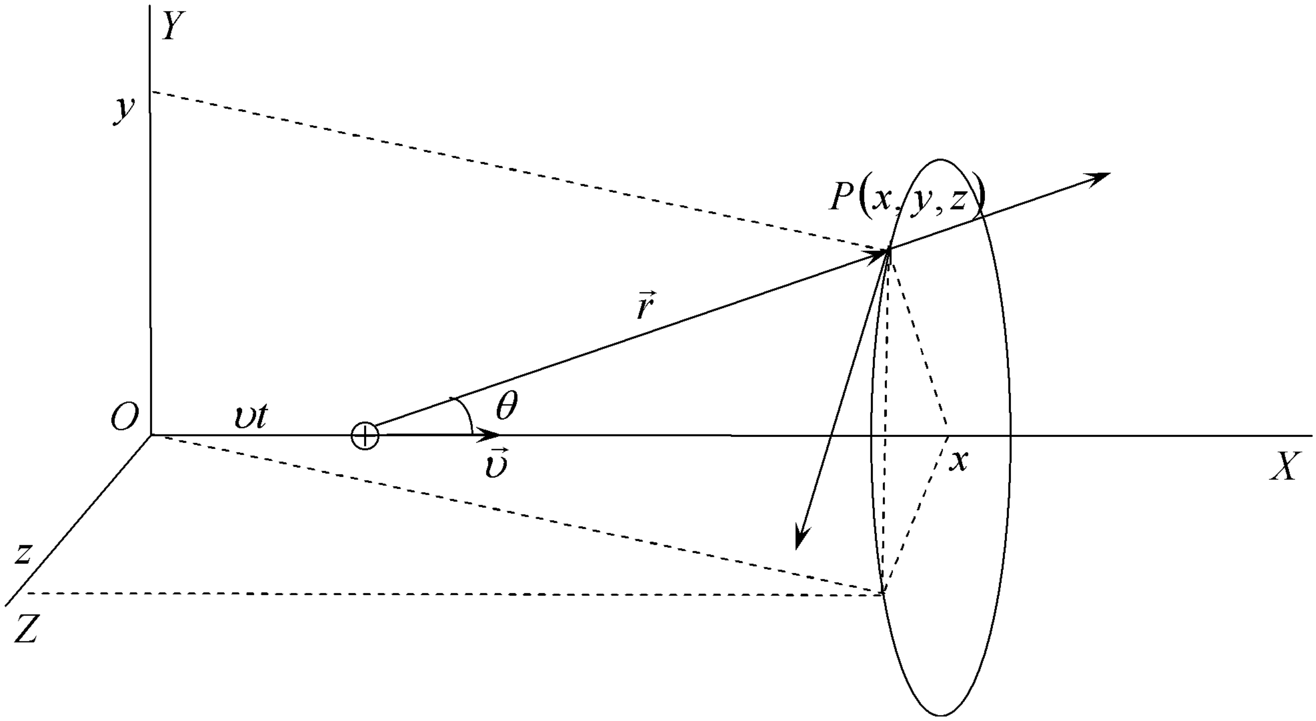

Figure 14: The electromagnetic field of a charged particle moving uniformly and in a straight line.

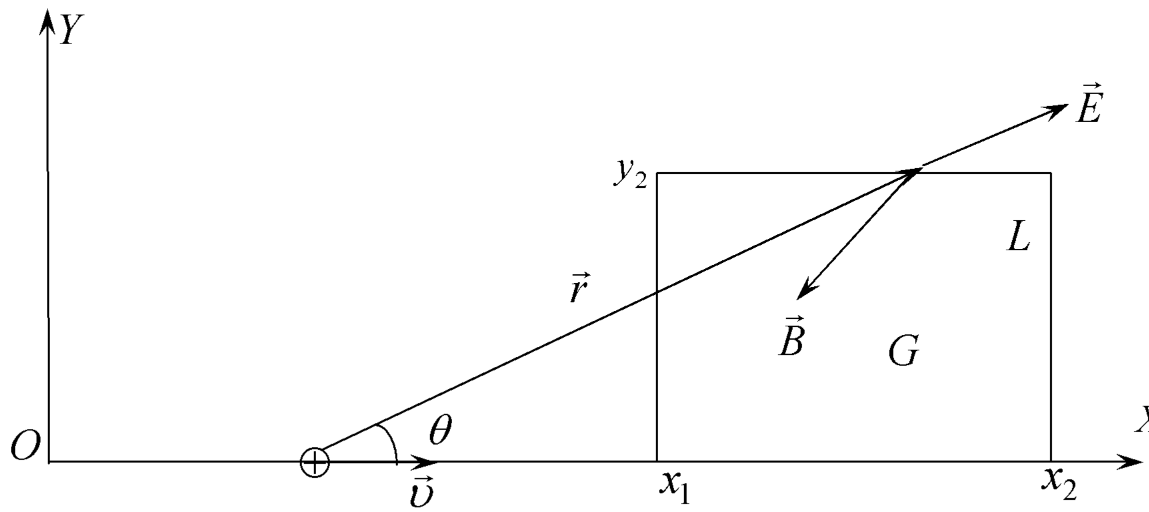

Figure 15: A conducting rectangular contour L is located in the XOY plane.

For the circulation of the vector $\mathbf{D}$ along the contour L (figure 15) we obtain [32, 33]

$$\oint_L \mathbf{D}\cdot d\mathbf{l} = \frac{q\beta^2}{4\pi}\left[\frac{1}{x_2 - vt} - \frac{1}{x_1 - vt}\right] + \frac{q\beta^2}{4\pi}\frac{1}{(x - vt)^2 + y_2^2(1-\beta^2)}\Bigg|_{x_1}^{x_2} \qquad (41)$$

That is, $\oint_L \mathbf{D}\cdot d\mathbf{l} \neq 0$ – the field of the vector $\mathbf{D}$ is non-potential. In other words, the rectangular conducting contour is in a non-potential electric field.

Therefore, the circulation of this non-potential field along a contour L stationary in the reference frame K is not equal to zero: $\varepsilon = \oint_L \mathbf{E}\cdot d\mathbf{l} \neq 0$ (see expression (41)).

That is, in any contour relative to which charged particles move (even at velocities

$v \ll c$), an EMF should arise, numerically equal to the circulation of the vector $\mathbf{E}$ of the non-potential field of the moving charged particle.

But this conclusion contradicts the principle of relativity, and the principle of relativity is one of the fundamental principles not only of physics but also of the modern worldview and methodology of scientific cognition.

Indeed, in the charged particle's own reference frame (K'), the contour L moves in the Coulomb potential field of the stationary charged particle. In this case, the EMF in the contour $\varepsilon' = \oint_L \mathbf{E}' \cdot d\mathbf{l} = 0$. According to the principle of relativity, the EMF in this contour should be equal to zero in any other reference frame.

Therefore, to satisfy the principle of relativity, we must assume the existence in the reference frame K of an additional EMF that compensates at any moment in time for the circulation of the non-potential field $\oint_L \mathbf{E} \cdot d\mathbf{l}$ in the contour L.

Example 1.4. A charged particle moves in a vacuum with velocity $\mathbf{v}$ in the reference frame K. Show that the non-potential electric field of the moving charged particle at each point in space and at any moment in time in the reference frame K is compensated by a vortex electric field generated by the variable magnetic field of this same moving charged particle.

Solution: Since the electromagnetic field of the moving charged particle is determined by formulas (5) and (12), the curl of the electric field intensity vector of the moving charged particle is equal to

$$\nabla \times \mathbf{E} = \frac{3q\beta^2(1-\beta^2)}{4\pi\varepsilon_0[(x-vt)^2 + (y^2+z^2)(1-\beta^2)]^{5/2}}[(x-vt)z\mathbf{j} - (x-vt)y\mathbf{k}]$$

and the partial derivative of the vector $\mathbf{B}$ [22]

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{3q\beta^2(1-\beta^2)}{4\pi\varepsilon_0[(x-vt)^2 + (y^2+z^2)(1-\beta^2)]^{5/2}}[(x-vt)z\mathbf{j} - (x-vt)y\mathbf{k}]$$

Comparing these last two expressions, we obtain the fundamental equation of the law of electromagnetic induction

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Thus, analysis of the properties of the electromagnetic field of a moving charged particle leads to an unambiguous answer: the non-potential vortex electric field of the moving charged particle at each point in space, at each moment in time, is compensated by the vortex electric field generated by the variable magnetic field, $\frac{\partial \mathbf{B}}{\partial t}$ [22, 27]:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{42}$$

Let us emphasise once again that to satisfy the principle of relativity, it is necessary to require that in the reference frame associated with the contour L (figure 14, figure 15), the circulation of the vector $\mathbf{E}$ along this contour is fully compensated by the EMF caused by the change in magnetic flux through the surface bounded by the contour L. That is, from the point of view of both reference frame K and reference frame K', no EMF should arise in the contour L.

It is precisely this requirement of the principle of relativity and the non-potentiality of the electric field of a moving charged particle that unambiguously lead to equation (42). This allows us to conclude that *the law of electromagnetic induction (42) is a consequence of the principle of relativity and Coulomb's law.*

In other words, the phenomenon of electromagnetic induction "appears" in order to compensate for the appearance in the laboratory reference frame of the circulation of the vector $\mathbf{E}$ along any closed contour of the non-potential electric field of a moving charged particle. The emergence of an induced EMF according to (42) is necessary for the fulfilment of the principle of relativity.

For methodological purposes, let us verify that the EMF in the contour shown in figure 15, which is in the electromagnetic field of a moving charged particle, is determined by $\varepsilon_{\text{ind}} = -\frac{\partial\Phi}{\partial t}$.

Example 1.5. Show that when a charged particle moves, the change over time of the magnetic flux through the stationary contour L (figure 15) is equal to the negative of the circulation of the vector $\mathbf{E}$ of the electric field of the moving charged particle along the contour L.

Solution: Let's find the flux $\int\limits_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}$ through the surface bounded by the rectangular contour L (figure 15)

$$\int\limits_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S} = \frac{\partial}{\partial t}\int\limits_S \mathbf{B}\cdot d\mathbf{S}$$

where $\mathbf{B}(\mathbf{r},t)$ is the magnetic induction of the field of the moving charged particle at points on this surface.

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0 q}{4\pi}\frac{1-\beta^2}{[(x-vt)^2+(y^2+z^2)(1-\beta^2)]^{3/2}}[\mathbf{v}\times\mathbf{r}]$$

$$B_z = \frac{\mu_0 q(1-\beta^2)vr\sin\theta}{4\pi[(x-vt)^2+(y^2+z^2)(1-\beta^2)]^{3/2}} = \frac{\mu_0 q(1-\beta^2)v\sqrt{y^2}}{4\pi[(x-vt)^2+(y^2)(1-\beta^2)]^{3/2}}$$

where $\sin^2\theta = \frac{\rho^2}{r^2}$, see figure 15.

Since the equation of the surface S bounded by the contour L has the form $z = f(x,y) = 0$, the magnitude of the magnetic flux through this surface at any moment in time is equal to

$$\Phi = \int\limits_S \mathbf{B}\cdot d\mathbf{S} = \int\limits_S B_z dS = \int\limits_G B_z dxdy = \frac{\mu_0 q(1-\beta^2)v}{4\pi}\int\limits_{x_1}^{x_2} dx\int\limits_0^{y_2}\frac{ydy}{[(x-vt)^2+y^2(1-\beta^2)]^{3/2}}$$
$$= \frac{\mu_0 qv}{4\pi}\left[\ln\frac{x_2-vt}{x_1-vt} - \ln\frac{x_2-vt+\sqrt{(x_2-vt)^2+y^2(1-\beta^2)}}{x_1-vt+\sqrt{(x_1-vt)^2+y^2(1-\beta^2)}}\right]$$

Thus, for the magnitude of $-\partial\Phi/\partial t$ we obtain:

$$-\frac{\partial\Phi}{\partial t} = \frac{\mu_0 qv^2}{4\pi}\left(\frac{1}{x_2-vt} - \frac{1}{x_1-vt} + \frac{1}{\sqrt{(x_1-vt)^2+y_2^2(1-\beta^2)}} - \frac{1}{\sqrt{(x_2-vt)^2+y_2^2(1-\beta^2)}}\right)$$

which naturally coincides with expression (41) for vacuum.

It is important to emphasize that when using non-relativistic expressions for the field vectors $\mathbf{E}$ and $\mathbf{B}$ of the electromagnetic field of a moving CP:

$$\mathbf{E}_{nr} = \frac{q}{4\pi\varepsilon_0}\frac{\mathbf{r}}{r^3};\quad \mathbf{B}_{nr} = \frac{\mu_0 q\varepsilon_0}{4\pi\varepsilon_0}\frac{[\mathbf{v},\mathbf{r}]}{r^3} = \frac{1}{c^2}[\mathbf{v},\mathbf{E}_{nr}]$$

we obtain a result that contradicts the principle of relativity:

$$\nabla\times\mathbf{E}_{nr} = 0,\ \text{but}\ \frac{\partial\mathbf{B}_{nr}}{\partial t} \neq 0$$

where $\mathbf{r} = (x-vt)\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is the radius vector drawn from the instantaneous position of the CP to a given field point.

Indeed, for the situation shown in figure 15 we have:

$$\frac{\partial \mathbf{B}_{nr}}{\partial t} = \frac{\partial B_{nr_x}}{\partial t}\mathbf{i} + \frac{\partial B_{nr_y}}{\partial t}\mathbf{j} + \frac{\partial B_{nr_z}}{\partial t}\mathbf{k}$$

$$\frac{\partial B_{nr_z}}{\partial t} = \frac{3\mu_0 q v^2 (x - vt) y}{4\pi((x - vt)^2 + y^2 + z^2)^{5/2}}$$

And therefore

$$-\frac{\partial \Phi}{\partial t} = \frac{\mu_0 q \varepsilon_0 v^2}{4\pi\varepsilon_0}\left(\frac{1}{x_2 - vt} - \frac{1}{x_1 - vt} - \frac{1}{\sqrt{(x_2 - vt)^2 + y^2}} + \frac{1}{\sqrt{(x_1 - vt)^2 + y^2}}\right)$$

At the same time, the circulation of the vector $\mathbf{E}_{nr}$ along the contour L is equal to zero (since $\nabla \times \mathbf{E}_{nr} = 0$).

That is, from the point of view of the reference frame associated with the CP, the EMF in the contour is equal to zero, and from the point of view of the reference frame associated with the contour, the EMF is equal to $-\partial\Phi/\partial t$. An obvious violation of the principle of relativity.

Thus, when analyzing any electromagnetic phenomena to obtain correct consistent results, one should use the formulas and methods of relativity theory.

The fundamental equation $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ was derived by us as a result of analyzing a particular non-local electrodynamic problem. Generalizing (38) to the case of a variable magnetic field created by arbitrary external sources, we can state:

The local form of the law of electromagnetic induction follows as a consequence of Coulomb's law and the principle of relativity.

The equation $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ is derived by us for an arbitrary point in space where there is a variable magnetic field, *but for a point stationary in a given reference frame.*

If we complicate the problem slightly and consider the motion of an "observation point" in an arbitrary magnetic field $\mathbf{B}(\mathbf{r}, t)$ at velocity $\mathbf{V}$, then we find that [27]

$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt} \tag{43}$$

where $d\mathbf{B}/dt = \partial \mathbf{B}/\partial t + (\mathbf{V} \cdot \nabla)\mathbf{B}$ is the change in time of the vector $\mathbf{B}$ from the point of view of the reference frame K in the body (medium) moving at velocity $\mathbf{V}$ relative to the reference frame K.

When the "observation point" moves in the field $\mathbf{B}(\mathbf{r}, t)$, the substantial or total derivative $d\mathbf{B}/dt$ (as already noted) describes the change in time of the field $\mathbf{B}$ not only because this field is non-stationary, but also because the point moves through an inhomogeneous field.

That is, we can generalize law (43) to arbitrary magnetic fields $\mathbf{B}(\mathbf{r}, t)$, created by other sources, and not only by a moving CP.

When generalizing this result to time-varying magnetic fields of arbitrary nature and using Stokes' theorem and the superposition principle, we obtain an expression for the EMF of induction in a stationary contour, equation (37).

That is, if an "observation point" moves at velocity $\mathbf{V}$ in an inhomogeneous and non-stationary field $\mathbf{B}(\mathbf{r}, t)$, then for the total derivative of the magnetic field induction vector we must, according to (43), write:

$$\frac{d\mathbf{B}}{dt} = \frac{\partial \mathbf{B}}{\partial t} - \nabla \times [\mathbf{V}, \mathbf{B}] + (\mathbf{B}\nabla)\mathbf{V} + \mathbf{V} \cdot \nabla \cdot \mathbf{B} - \mathbf{B} \cdot \nabla \cdot \mathbf{V}$$

In these problems $\nabla \cdot \mathbf{B} = 0$ always, and when the body moves as a whole $\nabla \cdot \mathbf{V} = 0$ (for $\mathbf{V} = const$). In other words, $\nabla \cdot \mathbf{V} = 0$ means "incompressibility" of the body [40, p. 264].

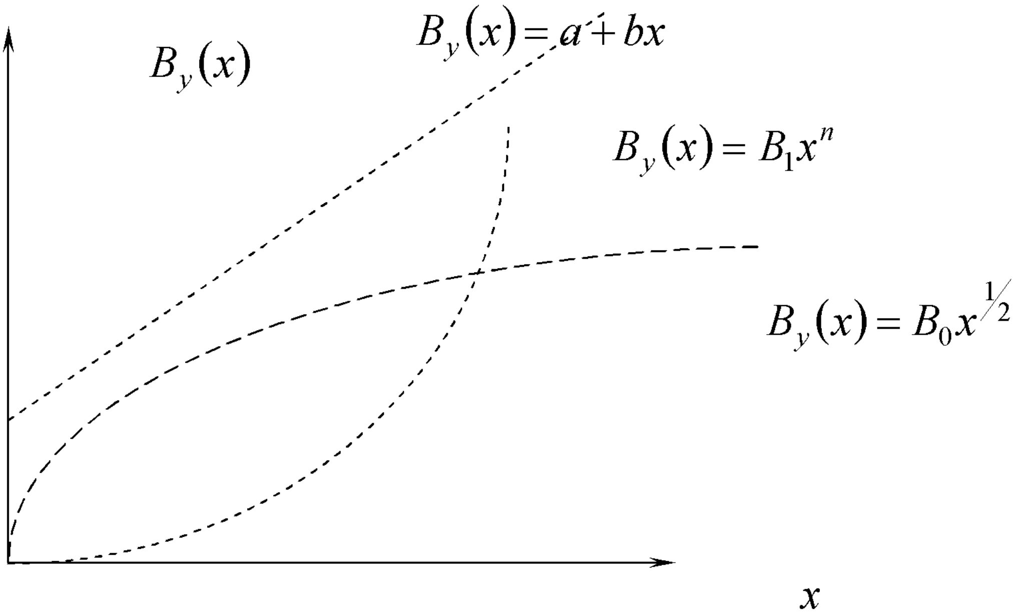

Figure 16: Illustration of the change in magnetic field over time when the "observation point" moves in stationary, inhomogeneous fields $B_y(x)$.

Therefore, the previous formula simplifies

$$\frac{d\mathbf{B}}{dt} = \frac{\partial \mathbf{B}}{\partial t} - \nabla \times [\mathbf{V}, \mathbf{B}] + (\mathbf{B}\nabla)\mathbf{V} \tag{44}$$

The term $(\mathbf{B}\nabla)\mathbf{V}$ takes into account the change in orientation of the vector $\mathbf{B}$ relative to the body [40, p. 264]. This term is equal to zero for translational motion with $\mathbf{V} = const$ and equals $[\omega, \mathbf{B}]$ for body rotation ($\mathbf{v} = [\omega, \mathbf{r}]$, where ω is the angular velocity).

That is, when moving at velocity $\mathbf{V}$ of the "observation point" in a magnetic field with induction $\mathbf{B}$, equation (43) takes the form:

$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt} = -\frac{\partial \mathbf{B}}{\partial t} - \nabla \times [\mathbf{V}, \mathbf{B}] \tag{45}$$

If the field is stationary, then $\partial \mathbf{B}/\partial t = 0$ and then the strength of the induction electric field in the conductor, according to (45), equals:

$$\mathbf{E} = \mathbf{V} \times \mathbf{B}$$

which is interpreted as the strength of the Lorentz force in traditional methods of studying the law of electromagnetic induction (see section 3.1 of the monograph [33]).

In the case of a simple distribution of the field in space, the meaning of the term $(\mathbf{V}\nabla)\mathbf{B}$ can be explained using figure 16, which shows stationary, inhomogeneous fields $B_y(x) = ax + b$, $B_y(x) = B_0\sqrt{x}$ and $B_y(x) = B_1 x^n$, where a, b, B_0, B_1 are some constants.

Moving or stationary contour are relative statements, and the characteristic of any vector field as constant or variable is generally relative.

For example, from the point of view of the reference frame K, the field B_y is constant but inhomogeneous, and from the point of view of the reference frame K', which moves at velocity $\mathbf{V} = const$ along the OX axis, the field $B_y(t')$ is a function of time (figure 16).

In existing methods of studying and explaining the electromagnetic induction phenomenon, attention is precisely focused on the emergence of EMF in stationary and moving contours in an arbitrary magnetic field. And then they wonder why different physical causes must be used to explain this phenomenon, despite the fact that the “flux rule” takes place in practically all cases.

These physical causes unite into one if the law of electromagnetic induction is written in the form $\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$, which follows from the principle of relativity and Coulomb’s law [22, 27, 32, 33].

And then, this mystery – the induced EMF in the contour – is always equal to the change in magnetic flux through the contour, as shown in [22, 27, 32, 33], disappears altogether if one looks more carefully at the law of electromagnetic induction (43) or (45).

In such fields, the magnetic field is observed as a time-varying field: $\frac{\partial B_y}{\partial t} = V\frac{\partial B_y}{\partial x}$.

Indeed, imagine a contour of any shape moving in an arbitrary magnetic field $\mathbf{B}(\mathbf{r}, t)$ at velocity $\mathbf{V}$.

Then, since the EMF is equal to the circulation of the vector $\mathbf{E}$, according to Stokes’ theorem, this circulation must be equal to the flux of the curl of the vector $\mathbf{E}$. And therefore, taking into account law (43), (45) we have:

$$\varepsilon_i = \oint\limits_L \mathbf{E} \cdot d\mathbf{l} = \int\limits_S \nabla \times \mathbf{E} \cdot d\mathbf{S} = -\int\limits_S \frac{d\mathbf{B}}{dt} \cdot d\mathbf{S} = -\frac{d\Phi}{dt} = -\frac{\partial \Phi}{\partial t} - \oint\limits_L [\mathbf{V}, \mathbf{B}] \cdot d\mathbf{l} \tag{46}$$

Thus, based on the generalized law of electromagnetic induction (43) we obtained law (46), which combines two physical causes of the occurrence of EMF of induction.

The vortex electric field is completely determined by the total time derivative of the magnetic field induction.

Thus, as a result of the conducted research, it is shown that the statement: “The relation $IR - \varepsilon = -d\Phi_m/dt$ is an independent law obtained experimentally, which cannot in any way be derived from the relations given earlier. Contrary to some assertions, the law of induction is not derived from the law of conservation of energy of the complete system of currents in a magnetic field” [52, p. 159] is erroneous.

That is, the principle of relativity underlies the unification of “two causes of the occurrence of EMF of induction”. At the phenomenological, phenomenal level, these two causes are due to the very process of cognition. The vortex electric field is generated only by the total time derivative of the magnetic field induction, $d\mathbf{B}/dt$.

The induction electric field in any case is non-potential as a whole, and the lines of the induction electric field strength vector are always closed.

Analyzing the model in which a segment of a conductor moves at velocity $\mathbf{V} = const$ in a magnetic field, the idea of the homogeneity of the induction electric field in this model is spontaneously formed. It is caused by the fact that in this case the analysis is carried out locally, without taking into account the picture of the electromagnetic field as a whole.

The induction current arising in the contour when it moves in an inhomogeneous, stationary magnetic field is caused by a vortex electric field, which is generated at each point of the contour (locally) by the Lorentz force field. And more precisely, the problem is described by equation (43) taking into account (44) (see figure 17).

Let’s show that the local form of the generalized law of electromagnetic induction (43) describes all those phenomena that in the traditional methodology of studying the phenomenon of electromagnetic induction are interpreted based on ideas about its dual nature.

Example 1.6. Let a contour be in the XOY plane of the reference frame K in a uniform magnetic field $\mathbf{B} = B_z\mathbf{k}$. The crosspiece AB moves at velocity $\mathbf{v} = v\mathbf{i}$ (figure 18). Find the strength of the induction electric field that arises at each point of the crosspiece.

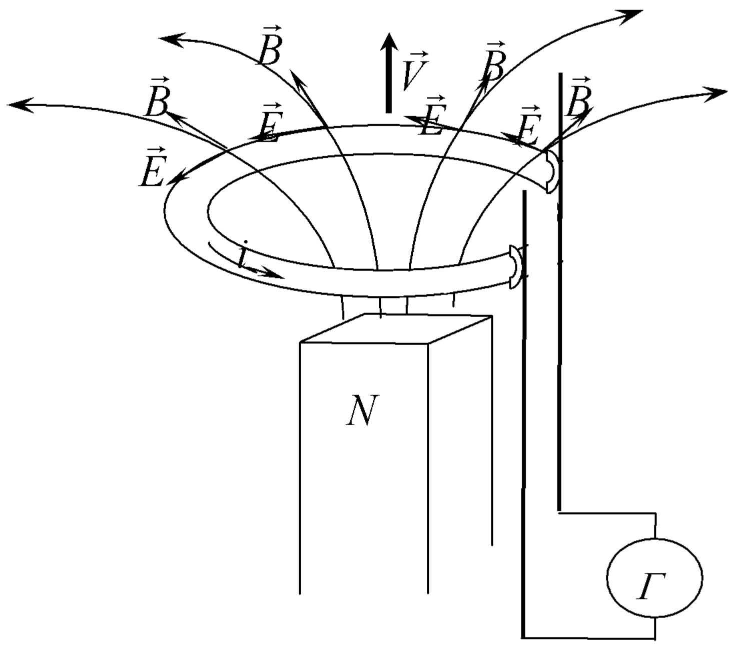

Figure 17: Vortex electric field generated at each point of the contour (locally) by the Lorentz force field.

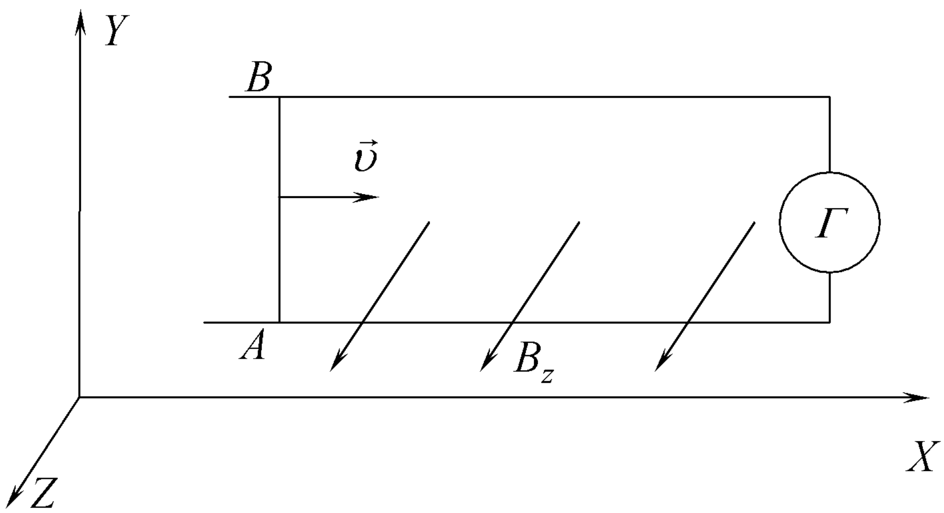

Figure 18: In a uniform magnetic field $B_z\mathbf{k}$, the crosspiece AB moves.

Solution: Since the magnetic field is stationary, the law of electromagnetic induction (43) for this case has the form

$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt} = -(\mathbf{v} \cdot \nabla)\mathbf{B} = \nabla \times [\mathbf{v}, \mathbf{B}]$$

That is, at each point of the crosspiece we have an induction electric field with strength

$$\mathbf{E} = [\mathbf{v}, \mathbf{B}], \quad \mathbf{E}_i = -vB_z\mathbf{j}$$

In this problem, according to the condition, the electric field at each point of the crosspiece is not vortex ($\nabla \times \mathbf{E} = 0$). This can be verified by direct calculation of the quantities $\nabla \times [\mathbf{v}, \mathbf{B}]$ or $-(\mathbf{v} \cdot \nabla)\mathbf{B}$.

Example 1.7. The geometry of the problem is the same as in Example 1.6, but the magnetic field is inhomogeneous, $\mathbf{B} = B_z(x)\mathbf{k} = B_1x^2\mathbf{k}$, where B_1 is some constant.

Find the strength of the induction electric field that arises at each point of the crosspiece.

Solution: Let's use the law of electromagnetic induction (43)

$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt} = -(\mathbf{v} \cdot \nabla)\mathbf{B} = \nabla \times [\mathbf{v}, \mathbf{B}]$$

First of all, let's verify by direct calculation that

$$-(\mathbf{v} \cdot \nabla)\mathbf{B} = \nabla \times [\mathbf{v}, \mathbf{B}].$$

Indeed, in this example

$$(\mathbf{v} \cdot \nabla)\mathbf{B} = v_x \frac{\partial \mathbf{B}}{\partial x} + v_y \frac{\partial \mathbf{B}}{\partial y} + v_z \frac{\partial \mathbf{B}}{\partial z} = v \frac{\partial B_z}{\partial x}\mathbf{k} = 2vB_1 x\mathbf{k}$$

$$[\mathbf{v}, \mathbf{B}] = -vB_z\mathbf{j} = -vB_1x^2\mathbf{j}$$

$$\nabla \times [\mathbf{v}, \mathbf{B}] = -\left[-\mathbf{i}\frac{\partial}{\partial z}vB_z + \mathbf{k}\frac{\partial}{\partial x}vB_z\right] = -\mathbf{k}\frac{\partial}{\partial x}vB_z = -2vB_1x\mathbf{k}$$

Thus,

$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt} = -\frac{\partial \mathbf{B}}{\partial t} - (\mathbf{v} \cdot \nabla)\mathbf{B} = -(\mathbf{v} \cdot \nabla)\mathbf{B} = -2vB_1x\mathbf{k}$$

Due to the geometry of the problem, the last differential equation takes the form

$$\nabla \times \mathbf{E} = \frac{\partial E_y}{\partial x} = -2vB_1x\mathbf{k}$$

$$dE_y = -2\mathbf{k}vB_1xdx$$

$$E_y = -vB_1x^2 + C = -vB_1x^2$$

We obtain the well-known result from school and general physics courses: the strength of the extraneous electric field is determined by the Lorentz force and equals

$$\mathbf{E} = [\mathbf{v}, \mathbf{B}(x)] = -vB_1x^2\mathbf{j}$$

Thus, the analysis of the study of the phenomenon of electromagnetic induction and the current state of its interpretation revealed ambiguity and lack of clarity in most approaches to its formulation. This situation is a consequence of the established approach in the scientific and methodological literature to the analysis of a significant part of educational models of electrodynamics within the framework of classical mechanics, despite the fact that electrodynamics in its essence belongs to relativistic physics. On the other hand, the section shows on what basis the law of electromagnetic induction itself should not be considered experimental.

Based on the analysis carried out, we have implemented an approach that makes it possible to obtain the law of electromagnetic induction as a consequence of the principle of relativity and Coulomb's law. The proposed problems, illustrative figures and the method of substantiating the law of electromagnetic induction should serve as the basis for the methodology of teaching electrodynamics at a pedagogical university.

8. The system of Maxwell's equations in vacuum as a consequence of the principle of relativity and Coulomb's law

In the methodological approach proposed in the previous sections of this paper for studying electrodynamics using STR methods, we have substantiated a number of basic laws and relations of classical electrodynamics.

Thus, from the analysis of the interaction of two moving CPs we obtained:

1. Expression for the magnetic field induction of a uniformly moving CP (12);
2. Biot-Savart law in relativistic form (17);
3. Expression for the Lorentz force (10) and (11);
4. Ampère's formula (15).

Application of the Biot-Savart law in relativistic form makes it possible to find:

5. Magnetic field induction of an infinitely long straight conductor with steady current (19);
6. Magnetic field induction of a segment of a straight conductor with steady current (20).

Moreover, the analysis of formulas (12) (20) as well as the Biot-Savart law itself in relativistic form (17) for velocities of charged particles motion $v \ll c$ can be considered as an illustration of the correspondence principle.

7. Within the framework of the proposed methodological approach, the system of Maxwell's equations in vacuum is relatively easily substantiated.

We obtain the first pair of Maxwell's equations from the analysis of the properties of the electromagnetic field of a uniformly moving CP (as the main model object in the proposed methodological system of teaching electrodynamics based on STR).

The local form of the law of electromagnetic induction follows as a consequence of Coulomb's law and the principle of relativity

$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

The equation $\nabla \cdot \mathbf{B} = 0$ is confirmed when finding the divergence of the magnetic field induction vector $\mathbf{B}$, created (generated) by any distribution of currents, or moving CPs (for example, $\nabla \cdot \mathbf{B} = 0$, where $\mathbf{B}$ is determined by (12)).

The first Maxwell equation of the second pair of ME is substantiated based on the following considerations. Since for both a stationary and a uniformly moving charged particle with charge value q, the flux of the vector $\mathbf{E}$ through any closed surface is determined by Gauss's theorem $\oint\limits_S \mathbf{E} \cdot d\mathbf{S} = q/\varepsilon_0$ [31, p. 19, p. 41-42], from this we obtain the first ME in local form:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

As for the second equation of the second pair of ME, as noted, the analysis of the properties of the electromagnetic field of a uniformly moving CP makes it possible to immediately write

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

where (25) holds for any stationary point in space "observation point" outside the dimensions of the moving charged particle.

And now let's find the curl of the magnetic field induction vector $\mathbf{B}$ of a moving CP in the case when the CP crosses the surface that rests on the selected contour.

First of all, note that according to (16) we have:

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \nabla \times [\mathbf{V}, \mathbf{E}] = \mu_0 \varepsilon_0 (\mathbf{V} \cdot \nabla)\mathbf{E}$$

The curl of the vector $[\mathbf{V}, \mathbf{E}]$ is found from the formula:

$$\frac{d\mathbf{E}}{dt} = \frac{\partial \mathbf{E}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{E} = \frac{\partial \mathbf{E}}{\partial t} + \nabla \times [\mathbf{V}, \mathbf{E}] + (\nabla \cdot \mathbf{E})\mathbf{V} - (\mathbf{V} \cdot \nabla)\mathbf{E}$$

But since for any point in space stationary relative to the reference frame K' (that is, stationary relative to the CP, the "observation point" moves at velocity $\mathbf{V}$ relative to the laboratory reference frame K) the total derivative $d\mathbf{E}/dt = 0$, therefore for the curl of the vector $[\mathbf{V}, \mathbf{E}]$ we obtain:

$$\nabla \times [\mathbf{V}, \mathbf{E}] = (\mathbf{V} \cdot \nabla)\mathbf{E} - (\nabla \cdot \mathbf{E})\mathbf{V} + \frac{\partial \mathbf{E}}{\partial t}$$

By the way, according to

$$\frac{d\mathbf{E}}{dt} = \frac{\partial \mathbf{E}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{E} = 0$$

for the partial derivative $\partial \mathbf{E}/\partial t$ we obtain (see also [31, p. 82-83]):

$$\frac{\partial \mathbf{E}}{\partial t} = -(\mathbf{V} \cdot \nabla)\mathbf{E}$$

This means that for an arbitrary point in space stationary in the reference frame K, the change in time of the vector $\mathbf{E}$ will be due to the "transport" of the electric field of the CP (1), which is used quite often in electrodynamic calculations [31, p. 41].

Now let's substantiate Maxwell's equation for the curl of the magnetic field induction vector using the properties of the electromagnetic field of a moving CP

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \nabla \times [\mathbf{V}, \mathbf{E}] = \mu_0 \varepsilon_0 \left[(\mathbf{V}\nabla)\mathbf{E} + \mathbf{V}\nabla \cdot \mathbf{E} - \mathbf{E}\nabla \cdot \mathbf{V} + \frac{\partial \mathbf{E}}{\partial t} \right]$$

Since $\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$, and $(\mathbf{V} \cdot \nabla)\mathbf{E} - (\nabla \cdot \mathbf{E})\mathbf{V} = 0$, then

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \left(\frac{\partial \mathbf{E}}{\partial t} + \mathbf{V}\frac{\rho}{\varepsilon_0} \right) = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{j}$$

where $\mathbf{j} = \rho \mathbf{V}$ is the current density caused by the motion at velocity $\mathbf{V}$ of a charge whose distribution in space is given by the charge density $\rho(\mathbf{r})$.

Thus, analyzing the properties of the electromagnetic field of a moving CP, we obtained Maxwell's equation

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{j} \tag{47}$$

Therefore, from a specific problem – analysis of the interaction of two moving CPs and analysis of the properties of the electromagnetic field of a uniformly moving CP – a set of laws and formulas of classical electrodynamics was obtained.

The validity of extending the laws obtained in this way (for one particular case) to arbitrary electrodynamic systems is based on:

a) using the superposition principle. That is, we consider that the electric and magnetic field of currents as well as the electromagnetic interaction of currents in general is a total (integral) manifestation of the interaction of moving charged particles;
b) experimental confirmation of the results of calculations using the formulas and laws obtained in this way.

Our approach is in line with generally accepted didactic ways of substantiating general laws from a result obtained in the analysis of a particular case.

Despite the fact that R. Feynman was quite skeptical about the idea that "all of electrodynamics can be deduced solely from the Lorentz transformation and Coulomb's law" [13, chap. 26], we believe that since the analysis of a particular electrodynamic example on consistent relativistic principles led to the basic laws of electrodynamics, then for an arbitrary electrodynamic situation the obtained results are valid.

All these laws and relations, strictly proven for a particular electrodynamic situation, can be used in the general case, relying both on practical confirmation and on the theorem on the uniqueness of the solution [61, p. 435-436].

The methods we proposed for substantiating the basic provisions of electrodynamics and Maxwell's equations should be considered not as an absolutely accurate proof of them, but as a certain didactic path, a methodological technique (method) for obtaining ME and other basic relations of electrodynamics.

Moreover, in our opinion, it is in principle impossible to prove absolutely accurately, in the most general form, for example, Maxwell's equations. There are, as already noted, several more or less general ways to substantiate Maxwell's equations. Each of them has its advantages over others, and its disadvantages.

The Maxwell equations obtained by us as a consequence of the principle of relativity and Coulomb's law can serve as a serious basis for constructing a methodology for teaching electrodynamics on a more coherent, less burdened with empirical facts and unsubstantiated provisions, theory.

Therefore, based on the research conducted, we have proposed the structure of the content component of the methodology for teaching electrodynamics based on the special theory of relativity (figure 19).

9. Use of computer models in teaching electrodynamics

Familiarizing students with methods of scientific research is one of the most important requirements of the principle of scientific character in studying physics. Among many methods of scientific cognition, an important place is occupied by the modeling method, which is used not only in physics, but also in many other branches of science.

Computer models easily fit into the structure of traditional forms of learning, allowing the teacher to model natural phenomena, create abstract models that in the process of studying electrodynamics were described verbally, or in a purely formal form.

Computer models are also an effective means of activating students' cognitive activity, which opens up wide opportunities for the teacher to improve the educational process.

In general, the application of the modeling method in the educational process is one of the topical issues of modern pedagogy and relevant methods. And this is quite natural, since the very process of knowledge formation is associated with the transformation in the student's mind of some models into others, which are derived from the first, but more accurate, with a greater approximation to physical reality [25].

The use of models for educational purposes helps to highlight and reflect the most important connections in phenomena that are often inaccessible to direct observation,

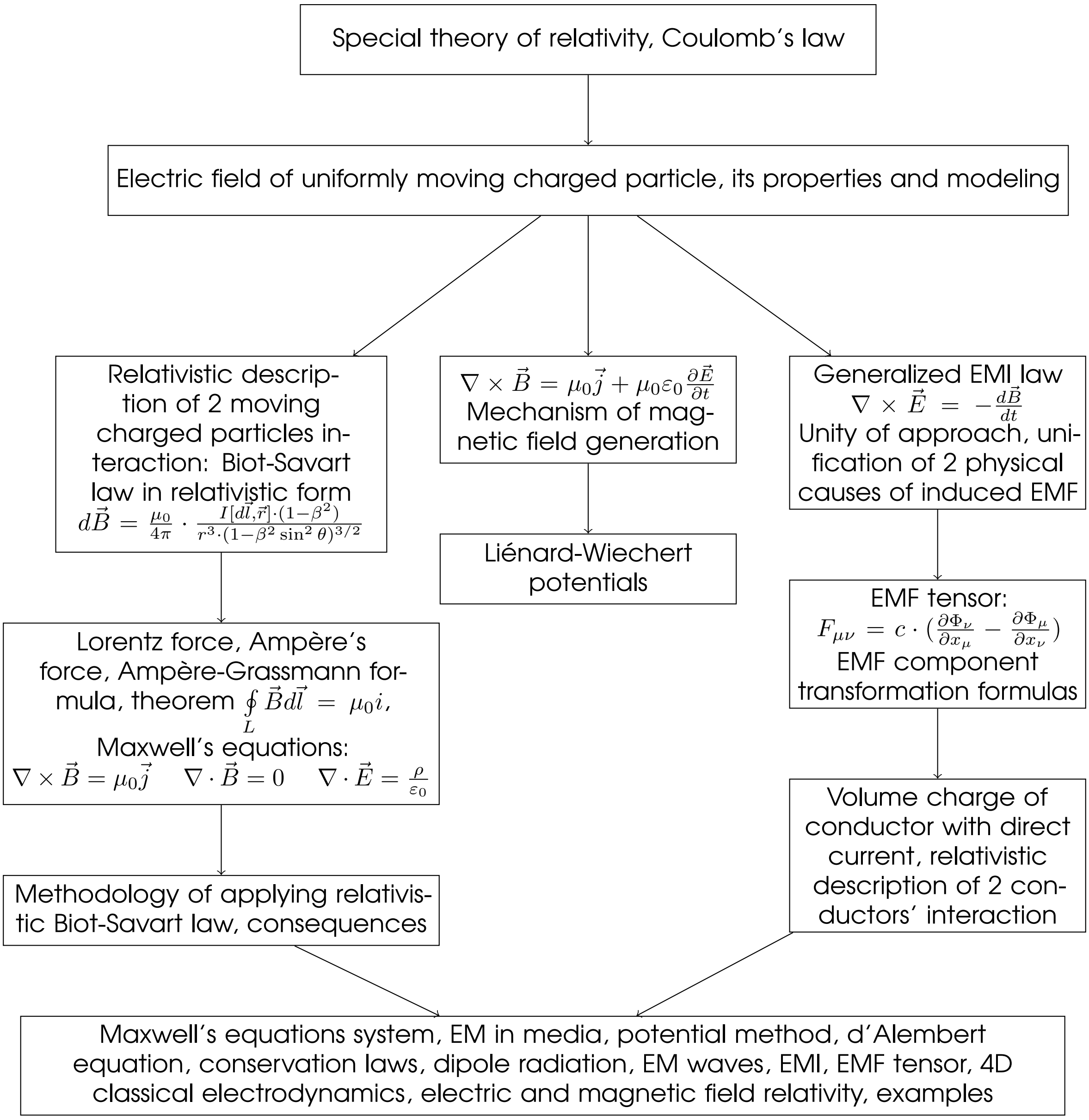

Figure 19: Structure of the content component for teaching electrodynamics based on special relativity theory.

to reveal the mechanism of the corresponding processes, to familiarize students with the experimental base of modern physics.

In addition to these didactic possibilities, the modeling method can also be used for independent work of students in studying the relevant sections of physics, and in particular, electrodynamics.

The method of mathematical modeling, which allows reducing the study of phenomena of the external world to mathematical problems, occupies a leading place among other research methods, especially in connection with the rapid development of computer technology. Mathematical models have also proven themselves as an important means of managing students' cognitive activity.

One of the possible directions of application of the mathematical modeling method (in particular) is the study of the relativity of electric and magnetic fields (see [35]). The program makes it possible to visually imagine and represent the electromagnetic field and the transformation of its components when transitioning from one reference frame to another, not only qualitatively - in the form of corresponding images, but also quantitatively, since the values of vectors $\vec{E}$ and $\vec{B}$ can be calculated with any accuracy for known fields $\vec{E}'$ and $\vec{B}'$ and the velocity of motion of the reference frame K' relative to the reference frame K. The developed software product also makes it possible to model the behavior of the components of the tensor f_{ik} of electric and magnetic fields in a medium, as well as the components of the polarization and magnetization tensor m_{ik}.

The use of this software product in the educational process, as experience in teaching students shows, provides opportunities for more thorough and visual illustration of the main consequences of the transformation formulas for electromagnetic field components.

In [38], the methodology for using the software product "Component", its capabilities for studying the phenomenon of relativity of electric and magnetic fields are described in detail.

For a better understanding of the mechanism of "flow" of displacement currents, and the distribution in space of the vector field $\vec{j}_{disp}$ and, thus, for a more complete explanation of the properties of the field $\vec{j}_{disp}$, we have created a simulation model that illustrates the distribution and change in space of the displacement current density of a moving CP, figures 20 and 21 [37].

With the help of computer simulation models, it is possible to demonstrate, and

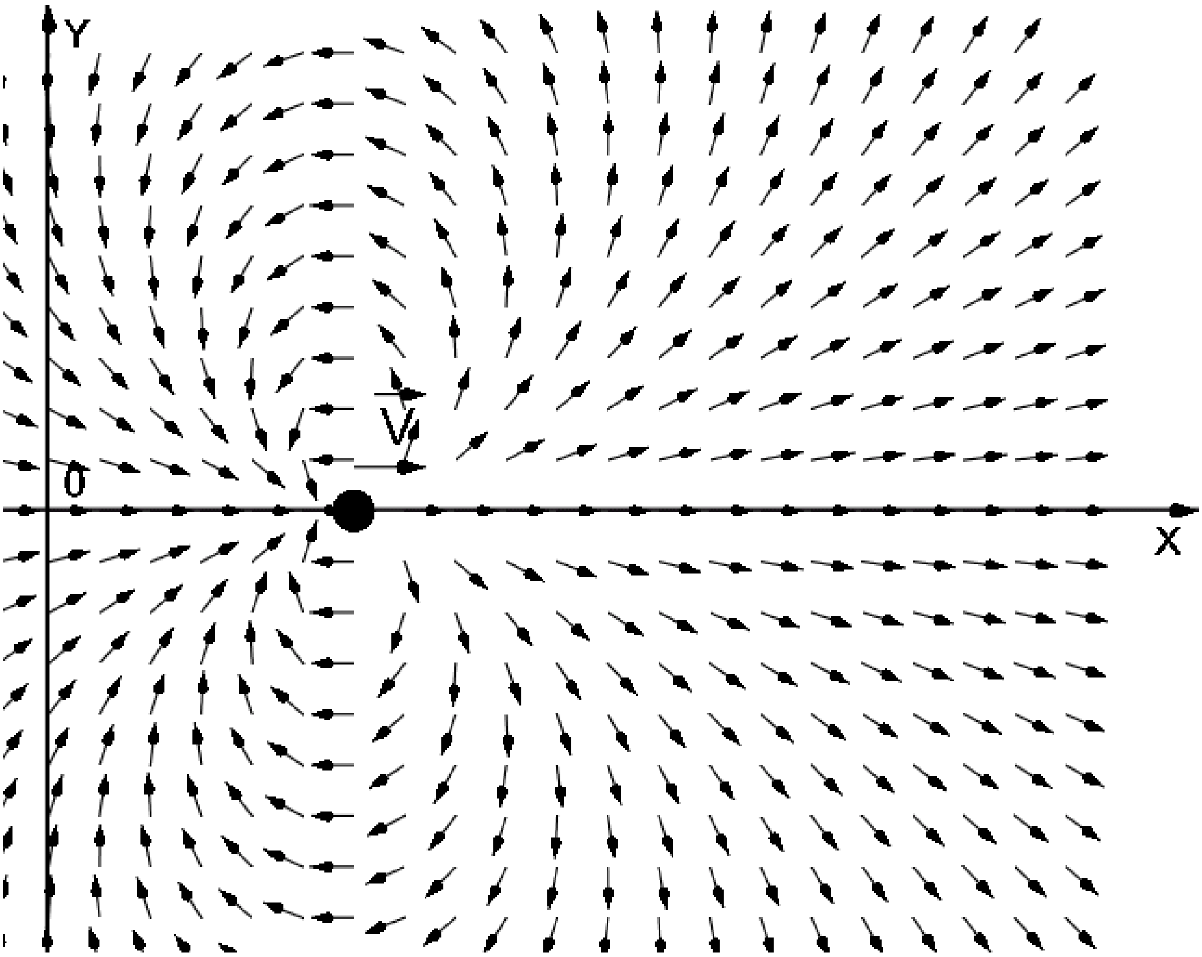

Figure 20: Distribution in the XOY plane of the displacement current density field $\vec{j}_{disp}$ of a charged particle moving uniformly and rectilinearly.

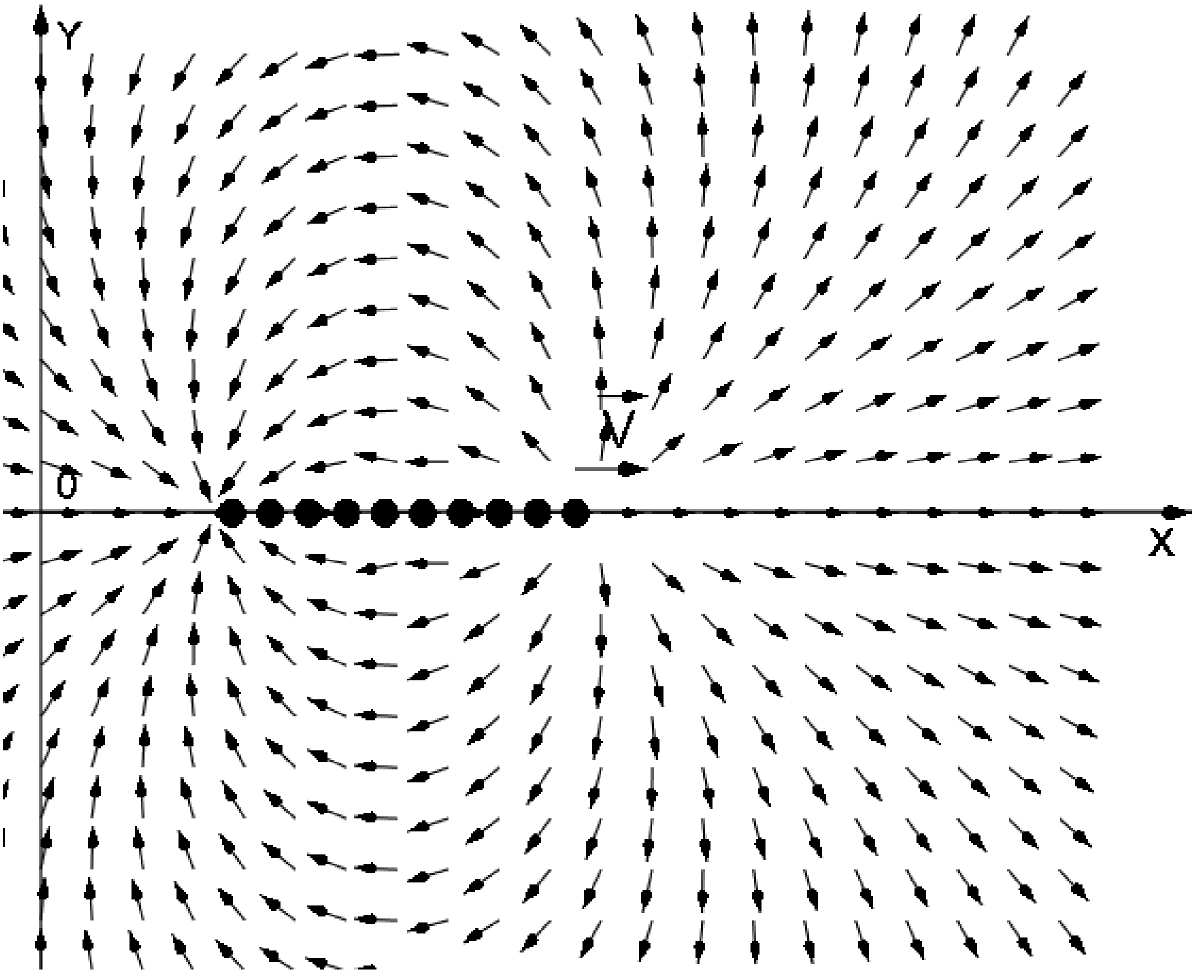

Figure 21: Distribution in the XOY plane of the displacement current density field $\vec{j}_{disp}$ during motion of a collection of charged particles with velocity $\vec{v}$ (37).

therefore imagine, within certain limits, the basic properties of the object being studied.

Thus, in particular, from figure 20 it can be seen that at any point in the plane perpendicular to the velocity vector of the CP motion $\vec{v}$, which the charged particle crosses at a given moment, the vector $\vec{j}_{disp}$ is directed opposite to the OX axis

$$\vec{j}_{disp} = -\frac{qv}{4\pi\rho^3\sqrt{1-\beta^2}}\vec{i}$$

The properties of displacement current and conduction current caused by a uniformly moving CP are described in more detail in [35].

To model the properties of the electromagnetic field of a uniformly moving charged particle, the Pascal programming language was used [50].

Using a computer model as a means of cognition, it is possible to illustrate the dependence of the electric field strength and magnetic field induction on the velocity of motion of the charged particle and the angle of observation.

Properties and computer modeling of the electromagnetic field of a uniformly moving charged particle.

1. The electric field strength of a moving CP depends on the direction of location of the field point (angle θ) and on the velocity of CP motion.

The magnitude of the electric field strength of a CP moving with an arbitrary velocity V decreases in the direction of motion and increases in the plane perpendicular to $\vec{V}$.

At relativistic velocities, the field strength of a moving charge at a given distance from it is small along the line of motion of the CP and large in the perpendicular direction, that is, the field seems to concentrate near the plane drawn through the instantaneous position of the CP perpendicular to its velocity.

In this regard, they say that the electric field of a moving CP is "flattened" in the direction of motion.

2. The electric field of a moving CP is not spherically symmetric, although it is characterized by significant symmetries, in particular the field of the vector $\vec{E}$ is mirror symmetric with respect to any plane passing through the OX axis (see the results of modeling the electric and electromagnetic field of a moving CP).
3. The electric field of a moving CP is, generally speaking, a non-potential field [22, 29, 36]:

$$\nabla \times \vec{E} \neq 0 \tag{48}$$

although one can indicate an infinite number of contours in this field, the circulation of the vector $\vec{E}$ along which is zero.

Thus, for example, $\oint\limits_L \vec{E} \cdot d\vec{l} = 0$:

a) along any contour lying in the YOZ plane passing through the instantaneous position of the CP;
b) along a symmetric contour, which, in turn, is symmetrically located relative to the YOZ plane passing through the instantaneous position of the CP;
c) along a symmetric contour that is symmetrically and perpendicularly located relative to any plane passing through the OX axis (for example, along a symmetric contour that is symmetrically located relative to the YOX plane.

That is, on the one hand, at any point of the field outside the CP the electric field is a vortex field $\nabla \times \vec{E} \neq 0$, on the other hand, it is potential (if the criterion of field potentiality is taken as $\oint_L \vec{E} \cdot d\vec{l} = 0$). Due to a certain symmetry of the electric field of a moving CP (figure 22), one can find forms of closed contours, the circulation $\oint_L \vec{E} \cdot d\vec{l}$ along which is zero.

But in essence, the field of the vector $\vec{E}$ of a moving CP is vortex, since, $-\nabla \times \vec{E}$ at any point of the field of a moving CP is completely determined by the rate of change in time of the vector of magnetic field induction of the moving CP $\partial\vec{B}/\partial t$ at the same point in space. Similarly, it can be shown that $\nabla \times \vec{H}$ is generated by $\partial\vec{D}/\partial t$.

In this regard, the question arises, how can the electric field, which is formed by the superposition of electric fields of a set of moving CPs (for example, the electric field of a conductor with steady current) be potential?! The solution to this contradiction is devoted to chapter 5 in [35].

4. For a better understanding of the properties of the electromagnetic field of a uniformly moving CP, we created a computer program for modeling and explaining the features of the electric field of a moving CP [29, 32, 33, 36] (for more details, see the author's monograph [33]).

The results of modeling the electric field of a moving CP turned out to be somewhat unexpected.

The picture of the electric field that we obtained (figure 22) is not described in existing electrodynamics textbooks. From the text of these textbooks, it follows that supposedly the electric field of a moving CP is flattened in the direction of motion in the same way as the Heaviside equipotential surfaces (see, for example, [42, p. 125], [56, p. 184]).

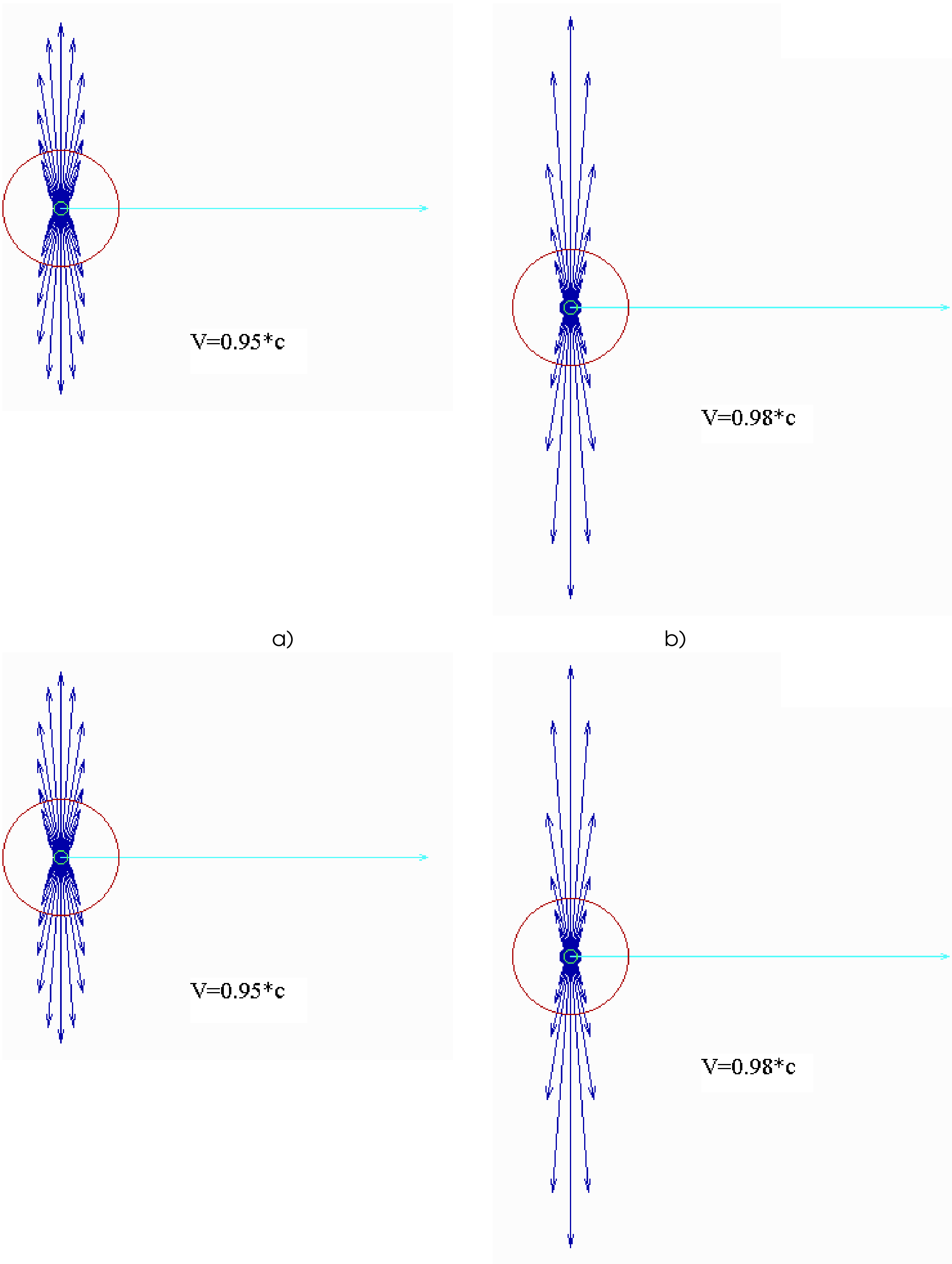

Figure 22: Polar diagrams of the electric field strength of a moving charged particle in the XOY plane at different values of particle velocity.

Computer modeling of the electric field of a moving CP and study of the function

$$f(\beta,\theta) = \frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}} \tag{49}$$

showed (figure 23) that the polar diagram of the electric field strength of a moving CP at high particle velocities does not have the form of a flattened ellipsoid.

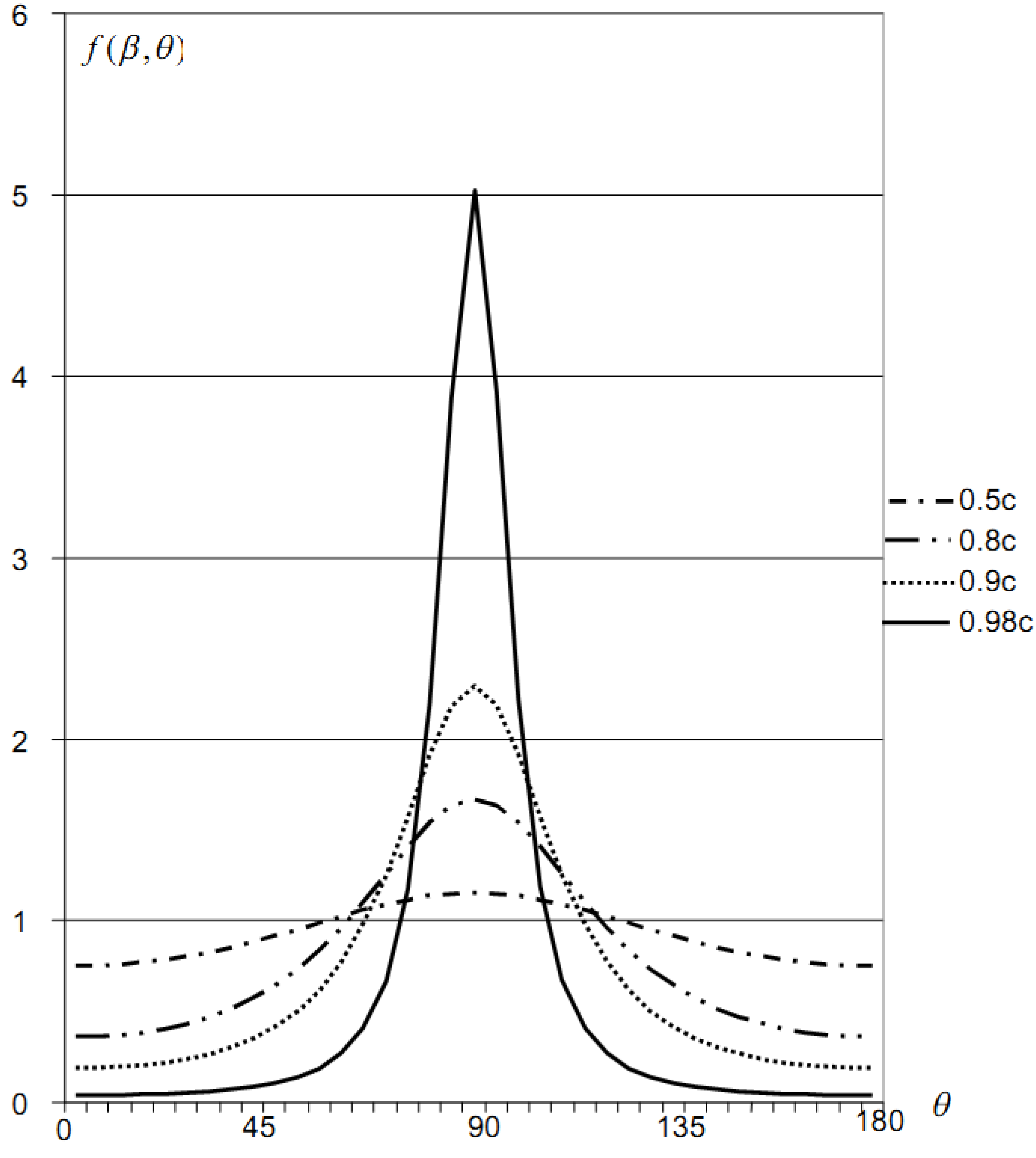

Figure 23: Dependence of the function $f(\beta,\theta) = \frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}$ on the angle θ at different values of CP motion velocity.

At $V \to c$ in the region of angle values $\theta \approx 0$, a kind of "constriction" is observed (see figure 22). On the polar diagram of the electric field strength and in the XOY plane, the polar diagram has the form of a "figure eight" (figures 22b, 22c, 22d).

Thus, analysis of formula (5) and computer modeling of the electric field of a moving CP showed that the "flattening" of the electric and magnetic fields in the direction of motion significantly depends on the angle θ. Namely, at $\theta \approx 0$ and $V > 0.95c$, $E_{||} \to 0$.

In other words, in any plane in which the OX axis lies, the magnitude of the vector $\vec{E}$ on the polar diagram forms a picture of the electric field resembling a "figure eight".

In the educational and methodological literature, there are no comments on such a picture of the electric field of a moving CP.

In addition, in the textbook [56, p. 163-164] and some methodological articles [4], the picture of the field presented using field lines not only unsuccessfully illustrates the properties of the electric field and its flattening in the direction of CP motion, but is also erroneous.

The picture of the electric field (5) in a three-dimensional model has the form shown in figures 24, 25, 26. In the center is a charged particle, and the distance from the center to points on the model is equal, on a certain scale, to the magnitude of the electric field strength at points in space equidistant from the instantaneous position of the CP.

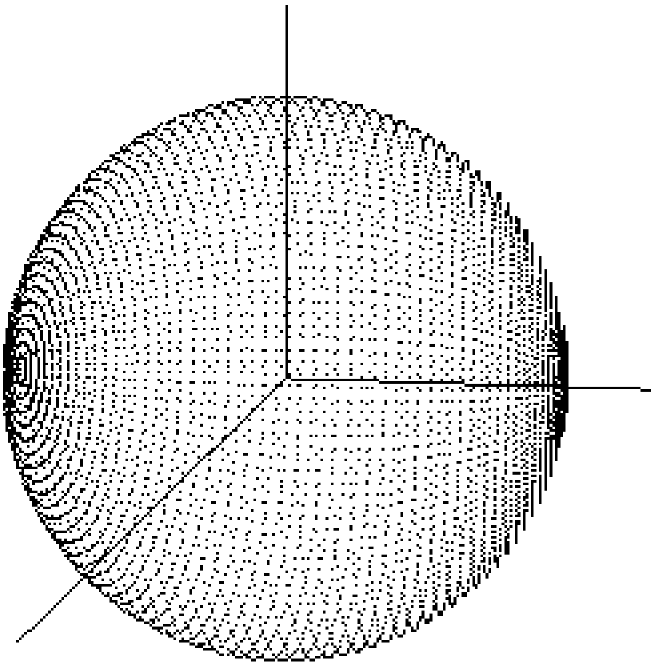

Figure 24: Field lines of the electric field of a charged particle moving with relativistic velocity.

Thus, at $V = 0$ we get a model that reflects the electric field of a stationary, or slowly moving ($V \ll c$), CP (figure 24).

From figure 24 it can be seen that the vectors of the electric field strength at the same distance from the charge are the same in magnitude. But this conclusion is true only at low velocities of CP motion, or when the CP is stationary.

As the velocity increases, the field of the charged particle seems to flatten in the direction of its motion. This is observed during the operation of the program, for example, $V = 0.6c$ (figure 25). On the left in figure 25 is a picture of the electric field in the XOY plane, on the right is a spatial picture.

At the velocity of CP motion ($V = 0.9c$) the picture of the field changes significantly (figure 26.

According to (15), $\vec{B} = \frac{1}{c^2}[\vec{V}, \vec{E}]$, the vectors of electric field strength and magnetic field induction are mutually perpendicular, besides, this is illustrated by the following figures, which reflect the orientation of vectors $\vec{E}$ and $\vec{B}$ in this model at velocities of charged particle motion $V \ll c$, $V = 0.6c$ and $V = 0.9c$, respectively (figure 27.

It can be seen that the $\vec{B}$ lines form concentric circles with a center on the OX axis, along which the charged particle q moves.

The configuration of the electromagnetic field of a charge moving uniformly and

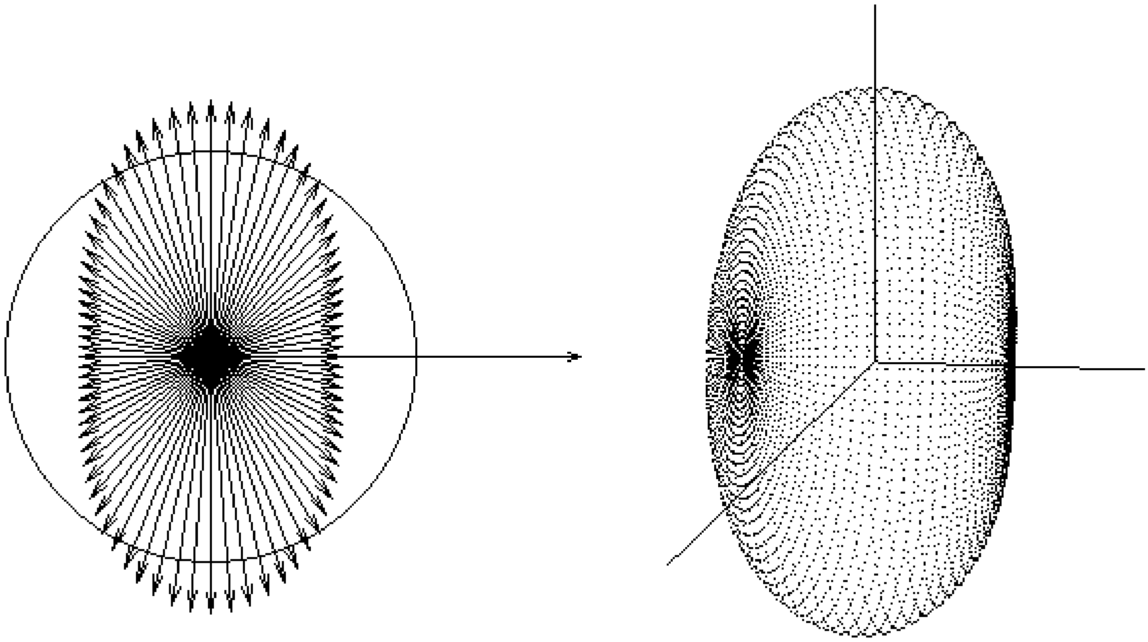

Figure 25: 2D and 3D models of the electric field of a moving CP at different velocities.

rectilinearly does not change over time, but only the position of this configuration changes relative to the fixed coordinate system K, that is, the unchanging field configuration moves together with the CP.

But if we fix an arbitrary point in space, then both the electric field strength $\vec{E}$ and the magnetic field induction $\vec{B}$ at this point are functions of time.

At the same time, one can observe how the appearance of the electromagnetic field changes as a whole, compare the magnitudes of the fields at different velocities of motion and in different directions relative to the direction of CP motion [36].

It is known that dynamic models more fully convey information about the properties of an object, and from a psychological point of view are better remembered, since the action of the model can be viewed and studied the necessary number of times, returning to different aspects of the mechanism, the temporal course of the phenomenon.

The experience of using our simulation models in the educational process has shown that they perform not only an explanatory function, but also contribute to a better understanding of the properties of the electromagnetic field of a moving CP and an in-depth study of the features of the electromagnetic field of a moving CP, thus implementing the principle of visibility in learning.

As the experience of using this program has shown, it is easy to perceive by both students and pupils.

In addition, as a result of modeling, we managed for the first time in the scientific and methodological literature to adequately depict the picture of the electric field of a moving CP and point out typical errors in the visual representation of this picture that occur in the scientific and methodological literature.

In our opinion, the created programs well illustrate the main properties of the electromagnetic field of a moving CP and can be recommended to pupils, students and physics teachers in educational institutions of all levels.

In order to more clearly present the absurdity of the statement about the experimental nature of the Biot-Savart law, we have created a computer model of possible

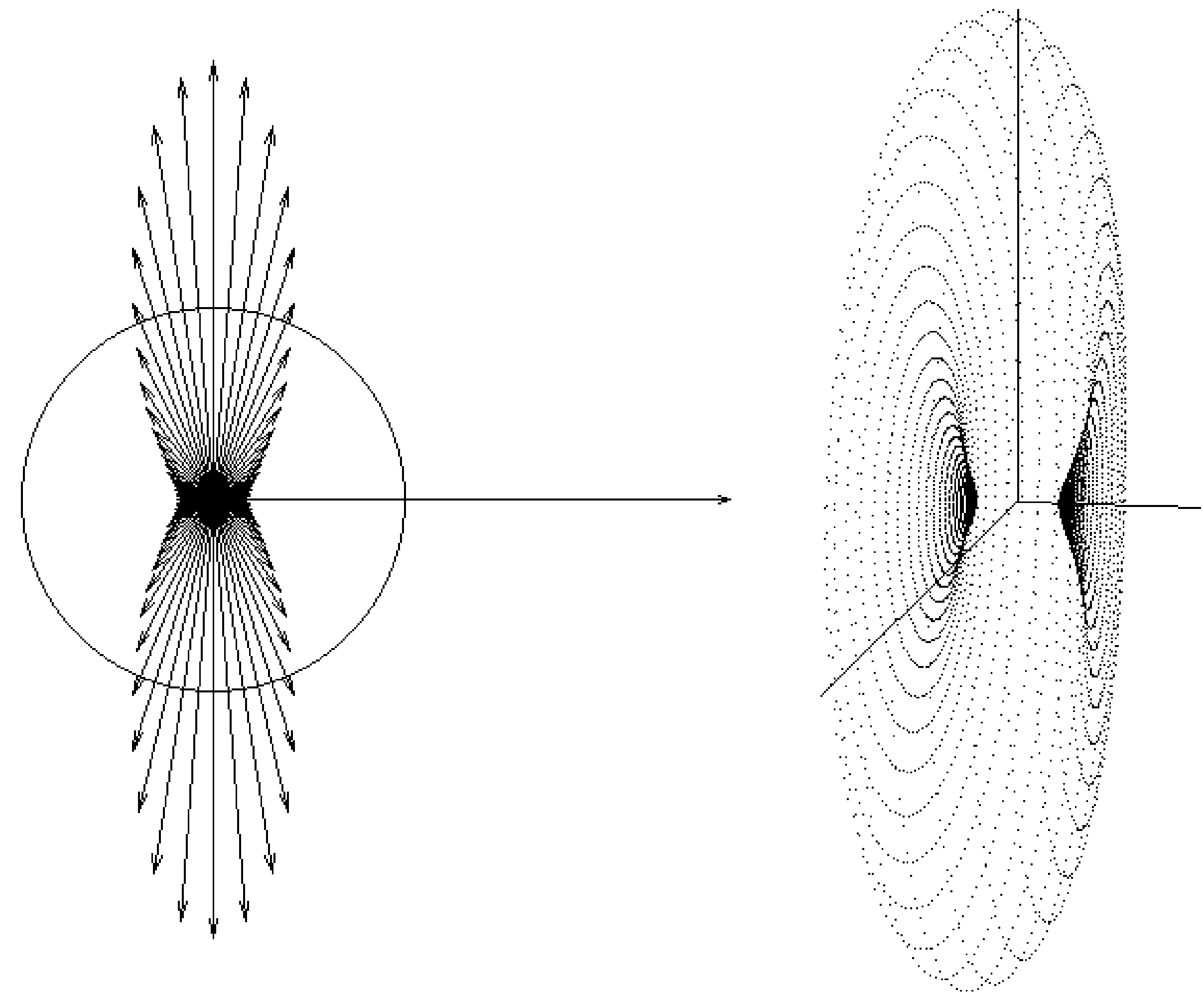

Figure 26: Electric field pattern of a moving charged particle at velocity $V = 0.9c$.

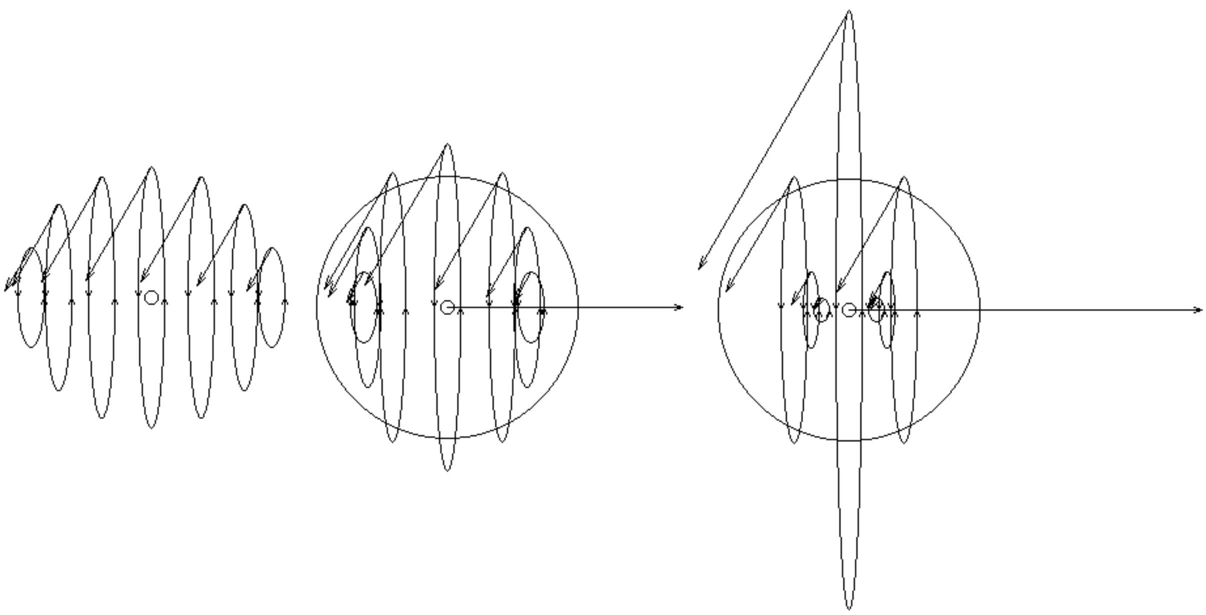

Figure 27: Dependence of the magnetic field induction $\vec{B}(\beta, \theta)$ of a moving CP on the velocity of CP motion $\beta = V/c$ and observation angle θ.

experiments by Biot and Savart. The computer program shows the true dependence of the magnetic field induction on the length of the steady current conductor l and on the distance of the field point from the steady current conductor, R (see formula (20))

$$B \approx \frac{\mu_0 i}{4\pi R} \frac{l}{\sqrt{\frac{l^2}{4} + R^2 \left(1 - \frac{V^2}{c^2}\right)}}$$

in comparison with the dependence that, according to historical sources, Biot and Savart obtained. Comparative analysis of the graphs of these two dependencies shows that only when $R \ll l$ do they coincide. Discussion and conclusions from these comparisons see in section 1.3 of chapter 1 in [35].

In the process of studying electrodynamics according to the methodology proposed by us, the heuristic significance of both thought experiment and modeling is demonstrated, since new scientific and methodological results are obtained, which confirm the thesis: "By model is understood … such a mentally imagined or materially implemented system that, reflecting or reproducing the object of research, is able to replace it in such a way that *its study gives new information about this object*" [32]. The results obtained in this way contribute to revealing the nature of physical phenomena (the relativistic nature of the magnetic field, the properties of the electric field of a conductor with steady current, finding a single principle underlying the "flux rule", the physical mechanism of generation of the magnetic field of steady currents, unipolar induction, the nature of the potentiality of the steady electric field of steady current).

Experience with these programs within the framework of the methodology of teaching electrodynamics as a relativistic theory and their implementation in the educational process in higher pedagogical educational institutions has shown their effectiveness.

10. Conclusions

1. Based on the principle of fundamentalization, a theoretically and methodologically grounded approach to structuring the content part of the course of theoretical physics (electrodynamics section) is substantiated. The content structure of the methodology for teaching electrodynamics based on the special theory of relativity is proposed.
2. The paper reveals the theoretical foundations of teaching electrodynamics to students of pedagogical universities based on relativity theory, the conceptual foundations of studying electrodynamics as a relativistic theory, and the interpretation of the basic laws of electrodynamics.
3. The conclusion is substantiated that in logical and methodological terms, the foundations of electrodynamics must be taught using consistently relativistic ideas and methods.
4. Based on a consistent analysis of the model of interaction of two charged particles moving at constant velocity (figure 8), taking into account the principles of special relativity theory, deductive and problem approaches, the following are substantiated:
   - formula for the Lorentz force;
   - expression for the magnetic field induction of a charged particle moving uniformly;
   - formula for the Ampère force;
   - Biot-Savart law in relativistic and classical forms;
   - law of electromagnetic induction.

5. The analysis made it possible to confidently consider formulas (10), (15), and the classical Biot-Savart law and the law of electromagnetic induction as not fundamental, since they turned out to be consequences of more general provisions.
6. It is shown that for the correct description of electromagnetic phenomena, it is necessary to take into account relativistic effects, even if they are infinitely small. Thus, the Biot-Savart law in relativistic form gives a correct description of physical phenomena, in particular for the electromagnetic field of rectilinear and uniform motion of charge carriers.
7. Based on the relativistic approach, the physical mechanism of generation of the magnetic field of steady currents in the space around a steady current conductor is proposed and described in detail.
8. Based on the principle of relativity and Coulomb's law, the phenomenon of electromagnetic induction is described. It is shown that based on the generalized law of electromagnetic induction, it becomes possible to describe those phenomena that in the traditional methodology of studying the phenomenon of electromagnetic induction are interpreted based on ideas about its dual nature.
9. A didactic approach is proposed for substantiating the basic provisions of electrodynamics and Maxwell's equations as a consequence of relativistic effects.

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A. Force of interaction between two uniformly charged filaments in reference frame K and in their own reference frame K'

Problem: Consider two parallel uniformly charged filaments. In their own reference frame K', the length of each is l', the charge density is τ', the distance between the filaments is d and they are oriented along the O'X' axis (figure 28). The reference frame K' moves with constant velocity $\vec{V}$ along the OX axis of reference frame K.

Find the force of interaction between these charged filaments in reference frame K.

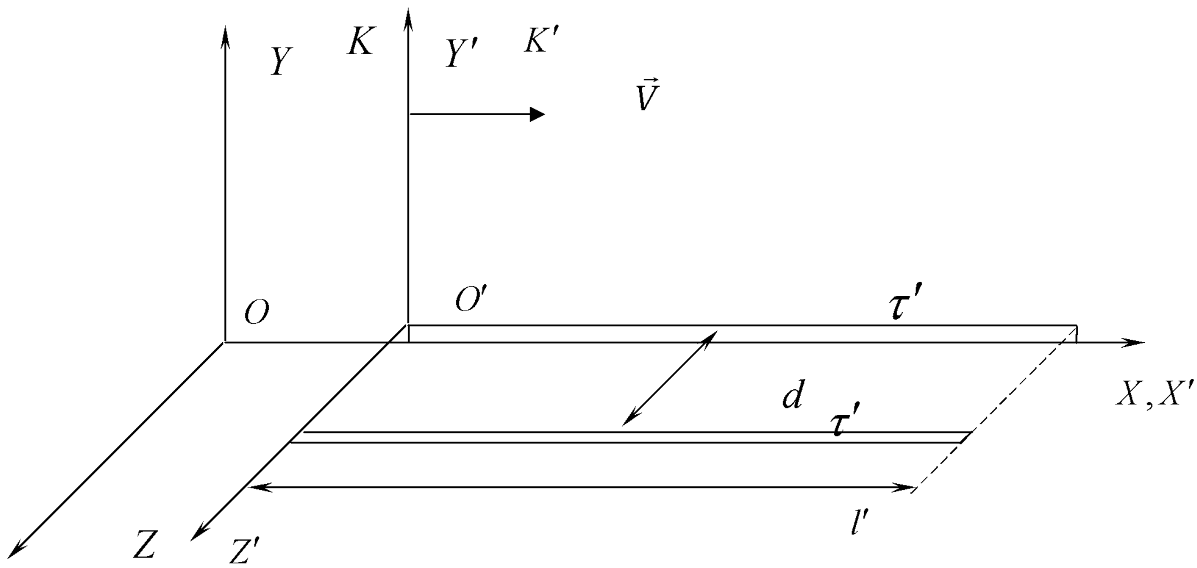

Figure 28: Interaction of two uniformly charged filaments.

Solution: For the interaction force F_z' in reference frame K' we have:

$$F_z' = \int_0^{l'} \tau' \cdot E_z' \cdot dx' = \frac{2\tau'^2}{4\pi\varepsilon_0 d}\left(\sqrt{d^2 + l'^2} - d\right) \tag{50}$$

where E_z' is determined by:

$$E_z' = \frac{\tau'}{4\pi\varepsilon_0 d}\left\{\frac{l' - x'}{\sqrt{(x' - l')^2 + d^2}} + \frac{x'}{\sqrt{x'^2 + d^2}}\right\}$$

In reference frame K, the resultant interaction force is equal to the geometric sum of the electric force and the magnetic force.

Note that according to the formulas for transforming force components when transitioning from reference frame K' to reference frame K, we have:

$$\gamma \cdot F_z = \gamma' \cdot F_z'$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\gamma' = \frac{1}{\sqrt{1-\beta'^2}}$, $\beta = \frac{v}{c}$, $\beta' = \frac{v'}{c}$, and v, v' are the velocities of the interacting bodies relative to reference frames K and K' respectively.

In this case $v' = 0$, so the relationship between F_z and F_z' should be:

$$F_z = F_z' \cdot \sqrt{1-\beta^2} \tag{51}$$

The electric and magnetic interaction forces between the filaments in reference frame K are equal to:

$$\begin{aligned} F_z^E &= \int\limits_0^{l'\sqrt{1-\beta^2}} \tau \cdot E_z \cdot dx = \frac{2\tau'^2}{4\pi\varepsilon_0 d\sqrt{1-\beta^2}}(\sqrt{d^2+l'^2}-d) \\ F_z^B &= \int\limits_0^{l'\sqrt{1-\beta^2}} \tau \cdot v \cdot B_y \cdot dx = \frac{2\tau'^2\mu_0 v^2}{4\pi d\sqrt{1-\beta^2}}(\sqrt{d^2+l'^2}-d) \end{aligned} \tag{52}$$

The total interaction force between the filaments: $F_z = F_z^E - F_z^B = F_z' \cdot \sqrt{1-\beta^2}$, *which is what the theory of relativity requires.*

Obviously, if we found the interaction force between the filaments based on the classical Biot-Savart law, requirement (51) would not be met.

B. Interaction between two conductors with direct current, whose models in traditional electrodynamics teaching methodology are unnaturally idealized

Problem: The model of two interacting conductors with direct current is shown in figure 29. Let in reference frame K the velocities of charged particles in them be equal to $\vec{v}$, the linear charge density in their own reference frame $|\tau_0^+| = |\tau_0^-| = \tau_0$. The distance between the conductors is equal to a. Find the interaction force between the conductors.

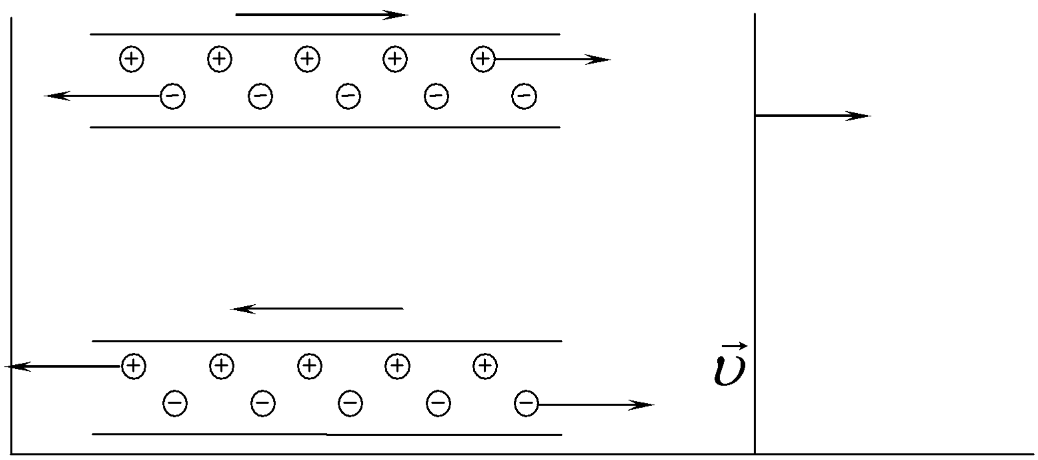

Figure 29: Models of two conductors with direct current, in which positive ions and electrons have equal in magnitude but oppositely directed velocities of motion.

Solution: To calculate the force acting on a unit length of one of these conductors from the other in the laboratory reference frame (LRF) K, it is necessary to calculate the force acting on positive charge carriers in their own reference frame and on negatively charged particles in their own reference frame. Then recalculate these force values according to the formulas for transforming force components [32, 33, 63] in reference frame K.

It is clear that the forces acting on positive and negative charge carriers of the first conductor in their own reference frames will have an electric nature. The electric field in the own reference frame of positive or negative charge carriers of the first conductor is due to the different magnitude of the linear density of positive and negative charges of the second conductor.

1st method: Then the force acting on the positive charges of the 1st conductor in their own reference frame, which are located on the same length as in reference frame K ($\Delta l' = \Delta l\sqrt{1-\beta^2}$, where $\beta = v/c$), from the electric field of the 2nd conductor is equal to:

$$F'_+ = \tau_0 \cdot \Delta l' \cdot E_2 = \tau_0 \cdot \Delta l' \cdot \frac{\tau_2}{2\pi\varepsilon_0 a} \tag{53}$$

where $\tau_2 = \tau_2^+ - \tau_2^- = \frac{\tau_0}{\sqrt{1-(\beta_2^+)^2}} - \tau_0 = \frac{2\beta^2\tau_0}{1-\beta^2}$ is the resultant linear charge density of the 2nd conductor from the point of view of the reference frame associated with the positive charge carriers of the 1st conductor; $\beta_2^+ = \frac{v_2^+}{c} = \frac{1}{c}\frac{v+v}{1+\beta^2}$; a is the distance between the currents.

Since we need to find the force acting on the length Δl of the conductor in reference frame K, we finally have, taking into account the expression for τ_2, and $\Delta l' = \Delta l\sqrt{1-\beta^2}$:

$$F'_+ = \frac{\tau_0^2\beta^2\Delta l}{\pi\varepsilon_0 a(1-\beta^2)^{3/2}} \tag{54}$$

The same magnitude (due to the symmetry of the problem) repulsive force F'_- acts on the negative charges of the 1st conductor in their own reference frame from the electric field of the 2nd conductor.

Since the reference frames associated with the charge carriers of the 1st conductor have a velocity v relative to reference frame K, the force acting on the segment Δl in reference frame K will be equal to (according to the formulas for transforming transverse components of force (20) [32, 33]):

$$F_y = F'_y\sqrt{1-\beta^2} = (F'_+ + F'_-)\sqrt{1-\beta^2} = \frac{\mu_0 I^2}{2\pi a}\Delta l \tag{55}$$

where the current $I = \frac{2\tau_0 v}{\sqrt{1-\beta^2}}$.

Thus, considering only electric forces, we found the usual expression for the force acting on the length Δl of one of them from the magnetic field of the other conductor with direct current.

2nd method: The same result can be obtained by finding the forces acting on moving chains of positive and negative charge carriers of the 1st conductor in the magnetic field created by the 2nd current. The electric field of the second conductor in reference frame K is absent ($E_2 = 0$), due to equal but oppositely directed velocities of negative and positive charge carriers.

The magnetic field induction created by the second current in reference frame K is equal to:

$$B = \int_{-\infty}^{\infty} |d\vec{B}| = \frac{\mu_0\tau_0 v}{\pi a\sqrt{1-\beta^2}} = \frac{\mu_0 I}{2\pi a} \downarrow \tag{56}$$

where $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I[d\vec{l}\times\vec{r}]}{r^3} \cdot \frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}$ is the Biot-Savart law in relativistic form [26]; here $\vec{r}$ is the radius vector drawn from the instantaneous position of the current element $\frac{\tau_0}{\sqrt{1-\beta^2}}dl$ to the given field point; θ is the angle between $\vec{v}$ and $\vec{r}$.

Then the force acting on a segment of length Δl of the first conductor from the magnetic field of the second conductor is equal to:

$$F_y = \frac{2\tau_0}{\sqrt{1-\beta^2}}\Delta l \cdot |[\vec{v}, \vec{B}]| = \frac{\mu_0 I^2}{2\pi a}\Delta l$$

which, naturally, coincides with expression (55).

Solving this problem in two ways is a good illustration of applying the principles of special relativity to calculate specific physical models. It was also shown that it is necessary to take into account relativistic corrections in the Biot-Savart law, and in the electric fields of moving charge carriers.