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A relativistic approach to teaching electrodynamics: Analysis of conductor interactions and relativistic foundations

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Abstract. This paper extends our relativistic framework for teaching electrodynamics in higher educational institutions. Building upon our previous work on deriving Maxwell's equations from first principles – the principle of relativity and Coulomb's law – we examine persistent contradictions in conventional electrodynamics teaching regarding conductors with constant current. We analyze the stationary electric field of current-carrying conductors, resolve contradictions concerning its potentiality, and explain the experimental non-observability of non-potential components through relativistic compensation effects. The paper addresses and resolves inconsistencies in the literature regarding the condition of neutrality for conductors with current, proposing a physically consistent condition: $\rho^0_+ = \rho^0_-$. Within this framework, we develop a relativistic description of the interaction between conductors with current that satisfies both the principle of relativity and physical adaptation requirements. This approach aligns with the fundamentalization of physics education, providing a theoretically robust alternative to traditional empirical methods of teaching electrodynamics. The proposed methodology creates a conceptually unified framework that better reflects modern physics while addressing existing inconsistencies in pedagogical literature, transforming how electromagnetism is taught in higher educational institutions.

Keywords: electrodynamics teaching, special theory of relativity, conductors with current, relativistic effects, electromagnetic interactions, current neutrality condition, physics education methodology, Maxwell's equations, fundamentalization of education

List of abbreviations

$\dot{f}$ – time derivative of function f
CCC – conductor with constant current
CET – classical electron theory
CP – charged particle
EF – electric field
EM – electromagnetic
EMF – electromagnetic field
EMI – electromagnetic induction
FR – frame of reference
FRS – frame of reference
GTR – general theory of relativity
HEI – higher educational institution
LFS – laboratory frame of reference

This article continues the discussion from "A relativistic approach to teaching electrodynamics: Deriving Maxwell's equations from first principles" [24], previously published in *Science Education Quarterly*, exploring further developments on the topic.

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MCP – moving charged particle
ME – Maxwell's equations
MF – magnetic field
PAP – principle of action principle
PEMI – phenomenon of electromagnetic induction
PR – principle of relativity
SC – system of coordinates
SEFCC – stationary electric field of constant current
SEI – secondary educational institution
STR – special theory of relativity
SWP – scientific worldview
TF – transformation formulas
TFCEFC – transformation formulas of components of electromagnetic field
UR – uncertainty relation
VEF – vortex electric field

1. Introduction

In connection with the intensification of Ukraine's European integration process, the secondary and higher physics education is undergoing systemic changes.

Reforms concern both the content of physics education and the forms and methods of teaching. The goal of reforming higher and secondary physics education is to create a system of physics specialist training that corresponds to the current state of science and technology, and which would enable physics graduates not only to work fruitfully but also to promote the development of creative abilities and self-realisation of the individual. The purpose of teaching physics in pedagogical higher educational institutions is for students to master fundamental scientific and professional knowledge, skills, and abilities necessary for professional activities at today's level of requirements, forming a physical way of thinking, and preparing specialists capable of working independently and creatively.

All this increases the requirements for the professional training of physics teachers and the realisation of their creative potential. One of the conceptual foundations for achieving these goals is the fundamentalisation of education.

We believe that the fundamentalisation of physics education means, first of all, the implementation of the didactic principle of scientific rigour; therefore, the technology of studying electrodynamics should correspond to the methodology of scientific knowledge.

Within the comparative analysis of electrodynamics teaching methodologies in pedagogical higher educational institutions and our proposed methodology for teaching electrodynamics based on the special theory of relativity, according to A. Einstein, we will call a law, principle, or experimental fact fundamental if it does not follow as a logical consequence from other provisions (physical principles, experiments) [35].

The fundamentalisation of physics education also means that the study of physics should be based on fundamental principles of physics and reflect, in particular, the dialectic of empirical and theoretical in the structure of physical knowledge, forming the worldview and physical way of thinking of the student.

Thus, based on just two fundamental provisions – the principle of relativity and Coulomb's law – we managed to substantiate the basic laws of electrodynamics (see [24]) and resolve contradictions in explaining some electrodynamic phenomena.

The purpose of physics as a scientific field is to study and analyse phenomena and processes of inanimate nature and methods of their research. At the same time, to identify and understand the fundamental causes underlying physical phenomena and to build a unified objective picture of the world is one of the main motives of physicists' activities.

However, the educational process is not identical to the process of scientific knowledge, just as the process of individual scientific knowledge is not identical to the process of socio-historical development of science. Therefore, adapted, pedagogically and didactically processed knowledge, methods of science, philosophical ideas, principles, laws, etc., are traditionally included in the content of education.

And if regarding this thesis there is no need to argue in building the content of the school physics course and even the course of general physics in higher educational institutions, the question of the structure and methodology of studying the course of "Theoretical Physics" still remains open.

Despite a significant increase in the role of scientific theory and methodology in the methodology of teaching physics, in most methodological manuals, both for general education and higher education, teaching is traditionally directed at considering an isolated section, mastering a set of phenomena, concepts, judgments, actions.

This approach is adopted at all levels and stages of studying the physics course, which becomes more complex and theorised, acquiring forms of separate clear theories, yet the study of which still traditionally occurs predominantly at the empirical inductive level, despite the fact that the process of forming physical concepts is internally heterogeneous and in teaching it is not necessary to adhere to the logic of unfolding the educational subject, according to which the empirical level always precedes the theoretical.

Honcharenko [9] noted on this matter: "...excessive adherence to the history of discoveries, rejection of proper theoretical analysis and absence of general methodology have led to the fact that today this course has essentially disintegrated into a set of separate special courses, little connected with each other. It is not only about the need to increase the volume of the physics course. A qualitative restructuring is needed, which would ensure the correspondence of physics as an educational discipline to today's logic and structure of physics as a science".

Quality training of a physics teacher is impossible without knowledge of fundamental physical principles and scientific theories. Leading didacticians emphasise the need for a close connection between the methodology of studying the discipline and the methodology of the basic science, because the essence of teaching is the method of thinking of the science. Electrodynamics and the special theory of relativity are fundamental physical theories that determine not only the level of relevant physics education but also the worldview and style of thinking of the specialist.

STR is a fundamental (general physical) relativistic conception of space-time, which, together with the laws and principles of quantum theory, lies at the foundation of the modern physical picture of the world. Relativistic ideas permeate all sections of physics, and electrodynamics itself is a relativistic theory. The significance of the philosophical and worldview potential of STR, its educational and upbringing function determine its universal human value as an integral element of culture not only for physicists by profession but also for modern educated people.

However, in the methodological plan, there is some alienation of the content of electrodynamics as an educational discipline, which is studied in higher pedagogical educational institutions, from relativistic physics.

It has already become a tradition to study electrodynamics as a science based on experimental laws (Biot-Savart law, Ampère-Grassmann law, law of electromagnetic induction), which, when studying, in a number of methodological works, are considered relatively isolated, or even completely unrelated; they are interpreted and considered as heterogeneous but fundamental facts; and meanwhile, all of electrodynamics is a relativistic theory.

It is enough to review the current educational programs in physics to make sure that when teaching electrodynamics in higher pedagogical educational institutions, the use of results and methods of the special theory of relativity is not provided, despite the fact that these sections of physics are organically connected with each other.

Traditional methods of teaching electrodynamics do not sufficiently use the basic provisions of the special theory of relativity. In this regard, in our opinion, the content, structure, and methodology of teaching electrodynamics do not correspond to the essence and methodology of this section of physics as a scientific field.

Obviously, the traditional approach is not devoid of significant advantages. Its strength lies in clarity, in reliance on seemingly obvious empirical facts that are difficult to deny, but at the same time leads to a certain dogmatism, to the idea of one-sidedness of physical knowledge with a hidden emphasis on empiricism. In this regard, Einstein [4] emphasised: "The prejudice — which has by no means died out in the meantime — consists in the faith that facts by themselves can and should yield scientific knowledge without free conceptual construction" [4, p. 13].

This scheme, reproducing the empirical path of knowledge, seems at first glance to be all-encompassing. At the same time, it does not reflect the multifaceted nature and diversity of learning in its content and organisational-procedural aspects. In particular, it diminishes the role of theoretical knowledge in learning.

Obviously, the inductive way of studying electrodynamics cannot promote the development of critical thinking, hinders psychological development and the formation of the physical style of thinking of the student and the pupil.

In addition, in some cases, traditional approaches in principle cannot explain the mechanism and nature of phenomena, but they give descriptions that are confirmed by experiments and experience, in the language of quantities that are experimentally directly measured: currents, voltages, active resistances, etc. And therefore, a significant number of physicists who deal with such devices and problems that are formulated in terms of exclusively phenomenological, electrical engineering quantities get the impression that physics is a purely experimental-research science.

This state of affairs not only does not correspond to modern requirements for the training of specialists in higher educational institutions but also distorts the notion of the modern scientific picture of the world.

Planck [34] and Bohr [2] believed that only theory can say what is measured or observed in an experiment, and physics without theory is not a science, but only a rather low-value conglomerate of separate facts, which is impossible to sort out.

That is, based on the principles of didactics, we believe that when a step has been made in science towards a deeper understanding of physical phenomena, then in the study and explanation of these physical phenomena in general education and higher educational institutions, this higher level of understanding and adequate interpretation should be reflected. Since classical electrodynamics is a Lorentz-covariant theory, from everything said above follows the conclusion that logically and methodologically, it is appropriate to teach the fundamentals of electrodynamics in the course of theoretical physics more purposefully, consistently using relativistic ideas and methods.

But there are attempts to study electrodynamics (electromagnetism) within the framework of the general physics course, starting from the system of Maxwell's equations. But this approach has problems of a methodological nature, related to the fact that it is not possible to substantiate Maxwell's equations at the very beginning of studying electrodynamics.

There are also attempts to study electrodynamics as a physical theory with the direct use of the provisions of the special theory of relativity. Thus, based on Lorentz transformations during the study of the magnetic field, the relativity of electric and magnetic fields and the relativistic nature of magnetism are shown (Feynman, Leighton and Sands [6], Matveev [27], Pinskij [33], Purcell and Morin [36], and others). This is a significant step forward.

But these authors solve only an insignificant part of the electromagnetism didactics issues. Moreover, the approaches they propose are not always consistent and correct.

In recent years, the methodological concept associated with strengthening the role of physical theories and theoretical generalisations in the study of physics has been relevant. This concept has not only not lost its significance even now – the problem of its implementation in teaching electrodynamics is even more acute than before.

Further development of Einstein's ideas can be found in the methodological approach of generalising knowledge around fundamental physical theories, introduced by V. G. Razumovsky and in the implementation of the more global principle of fundamentalisation of education [9].

The implementation of the more global principle of fundamentalisation of education [9] gives grounds to form not only separate knowledge but also to lay the foundations of the entire system of knowledge, to reveal internal connections between fundamental concepts and laws, to show their manifestation on specific facts and phenomena of reality. Principle of generalisation provides for the selection of one or several core ideas and grouping material around them.

In constructing an adequate methodology for the study of electrodynamics, we also proceeded from priority goals laid down in normative state documents of Ukraine about the need for thorough mastery of basic physical phenomena and ideas, mastering fundamental concepts, laws and theories of classical and modern physics, as well as methods of physical research, formation of scientific worldview and modern physical style of thinking in pupils and students are consonant with the methodological guidelines, scientific and methodological foundations of A. Einstein's creativity. And this means, in particular, that the use of the ideas of the special theory of relativity and their study are not just desirable, but necessary to achieve the goal called for by the reorganisation and reform of the content of physics education in Ukraine.

Therefore, the methodology of teaching electrodynamics in higher pedagogical educational institutions must be based on: the concept of holistic reflection of science in the educational process; the structure of knowledge, methodology of research of electrodynamic phenomena; didactic principles of teaching methodology in higher education; the idea of evolutionary transition from empiricism to broad theoretical generalisations using fundamental physical theories.

In our opinion, the characteristic tendency of the development of modern physics does not find full and adequate reflection in the process of teaching theoretical physics in higher pedagogical educational institutions, and in particular electrodynamics: relying on a small number of basic principles, to formulate and explain the entire set of physical phenomena and laws of the corresponding section of physics.

One of the components of A. Einstein's worldview was the conviction that the most adequate for physics is the hypothetical-deductive path of knowledge and learning, according to which in the process of learning physics it is necessary to formulate (construct, choose, make) these basic principles as simple and few as possible, without missing in this case an adequate presentation of anything contained in the relevant physical experiments.

"...It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience" [3, p. 165].

"The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them" [5, p. 44].

But at the same time, A. Einstein further states, "Nobody who has really gone deeply into the matter will deny that in practice the world of phenomena uniquely determines the theoretical system, in spite of the fact that there is no logical bridge between phenomena and their theoretical principles" [5, p. 44].

According to A. Einstein, in cognition and in teaching physics, there should be the aspiration in "developing unification of the logical structure, that is with the reductionin the number of the logically independent conceptual elements required for the basis of the whole system" [3, p. 165]

Regarding didactics of physics, this should mean, in our opinion, that when studying any section of physics, it is necessary to rely on a small number of fundamental physical laws.

In our opinion, the fundamentalisation of physics education in current conditions is the implementation of A. Einstein's ideas, according to which, concerning electrodynamics, we understand the creation of a scientific-methodical system of teaching electrodynamics, built on the basis of a small number of fundamental principles, from which then by the method of deduction, consequences are obtained that correspond to the totality of empirical data.

And the currently existing methods of studying electrodynamics are based on combining a large number of separate experimental facts in the form of empirical laws, from which by comparison, general laws are established.

In modern didactics, the need for an organic connection between the methodology of studying the discipline and the methodology of the basic science is emphasised.

Thus, an obvious need for scientific and methodological substantiation and creation of a new methodical system for studying the basic provisions of electrodynamics in higher pedagogical educational institutions, which would not have the above-mentioned shortcomings, was identified.

Our task is to substantiate, in the process of teaching electrodynamics, the main provisions of electromagnetism on the basis of the smallest number of fundamental principles (principle of relativity and Coulomb's law) along the shortest path, and to explain their physical essence, recognising from the very beginning the system of Maxwell's equations as a truth confirmed by experience.

In other words, our task is to implement A. Einstein's ideas regarding the substantiation of the basic provisions of electrodynamics, in the process of teaching it in higher pedagogical educational institutions, on the basis of as small as possible a number of logically independent from each other basic axioms and concepts.

The relevance of such a formulation of the problem is also conditioned by the inconsistency between:

- formal and insufficiently in-depth study of electrodynamics and the special theory of relativity not only in school but also in higher pedagogical educational institutions and their physically deep scientific content;
- secondary place of the special theory of relativity in physics courses of secondary and higher educational institutions and the fundamental importance of the theory of relativity in modern physics and, in particular, in electrodynamics;
- existing scientific and methodical training of physics teachers and the need for them to organise the process of studying electrodynamics by pupils in secondary educational institutions at the level of modern requirements;
- extremely deep combination of electrodynamics and the special theory of relativity in the formation of physical style of thinking and scientific worldview and formal, and sometimes unsatisfactory study of them.

Students get the impression that the special theory of relativity has no relation to everyday electrodynamic physical processes, because it is somehow believed that the effects of the special theory of relativity can significantly manifest themselves only when systems of reference or bodies move with speeds close to the speed of light in vacuum.

Such ideas do not correspond to physical reality and are methodically and methodologically harmful. The application of the principles of the special theory of relativity for the substantiation of the basic laws of classical and relativistic electrodynamics will demonstrate the "work" of the principles of the special theory of relativity and contribute to a deeper understanding of the essence of electrodynamics and the special theory of relativity.

Electrodynamics is a relativistic theory (for example, the d'Alembert equation together with the Lorentz calibration condition is covariant with respect to Lorentz transformations). And since electrodynamics studies the interaction between charged particles, currents, and the electromagnetic field, which is created by these and other charged particles, it studies the properties of the electromagnetic field, which is created by these and other charged particles, it is natural to assume that this covariance is laid in the essence and mechanisms of interaction of two moving charged particles.

Therefore, in this and previous [24] papers, we propose the following vision of the process of teaching electrodynamics: starting from Coulomb's law and the principle of relativity, as fundamental and independent of each other provisions, we will substantiate the basic laws of electrodynamics of constant and quasi-stationary currents, resolve contradictions and shortcomings of the "traditional" methodology of teaching electrodynamics, propose an explanation of the physical mechanism of generation of the magnetic field, substantiate the system of Maxwell's equations in vacuum.

In addition, a specific application of the principles of the special theory of relativity for the physical and methodical analysis of many electrodynamic problems is shown, in particular:

- a relativistic description of the interaction between conductors with currents is presented;
- in the model of a conductor with a constant current, which is widely used in physics didactics, an explanation of the experimental non-observability of the non-potential electric field of linear precession of charged particles is proposed;
- it is shown that the explanations available in the scientific and methodical literature of the relativistic causes of the appearance of "charge of a conductor with current" are contradictory and need clarification;
- the expediency of the proposed condition of neutrality of a conductor with a constant current is theoretically and methodically proven;
- the heuristic and didactic role of electrodynamic models in teaching electrodynamics is demonstrated.

2. Review of traditional, generally accepted ideas about the nature of the stationary electric field of a conductor with current

The study of what would seem to be a sufficiently transparent electromagnetic phenomenon – the flow of a constant current through a homogeneous metallic conductor (for simplicity and clarity, we will further consider a cylindrical and sufficiently long conductor) – does not hide any surprises and problems. And yet there are some physical questions related to this phenomenon that are not addressed in the process of teaching electromagnetism.

When studying the laws of constant current in higher and secondary schools, as a rule, they are limited to Ohm's laws and the consequences that follow from them. The electrotechnical level of teaching these issues inevitably leads to the fact that the nature and mechanism of the emergence of a stationary electric field in a conductor with current (SEFCC) are not discussed, or almost not discussed.

We are talking, firstly, about the mechanisms of the emergence of the electric field of a conductor with a constant current (CCC), secondly – about the nature of the volume charge that may arise in the CCC, and thirdly, about the electromagnetic interaction between conductors with currents.

We can point to three physical phenomena which, in principle, lead to the emergence of an electric field both inside and outside the CCC:

1. SEFCC is created by surface charges distributed in a certain way on the surface of a cylindrical conductor with current.
2. The electric field of the CCC can be created by a volume charge that appears as a result of the pinch effect (section 4).
3. An additional electric field of the CCC can be created by charges caused by the difference in the values of charge densities of the set of electrons and ions of the crystal lattice due to their movement with different velocities in some FR (section 4).

Let's first focus on the analysis of the first physical phenomenon.

It can be considered that in the scientific and methodical literature, the *conclusion that charged particles that create a uniform stationary electric field in a cylindrical conductor with current are located on the surface of the conductor is sufficiently substantiated* [11, 25, 28, 30, 31, 37, 39].

But regarding the question about the nature of SEFCC and the state of motion of these surface charges, there are different points of view in the educational and methodical literature. Indeed, let's cite several statements on the essence of this issue, taken from well-known manuals.

"Stationary EPPCC inside the conductor is created by *immobile, constant in time surface charges. In this respect, EPPCC by its nature is a Coulomb field – it is analogous to the electrostatic field of immobile charges, and has a potential character*" [25, p. 111] (emphasis in quotes is mine – O. K.).

That is, it is only analogous to the Coulomb (electrostatic) field, but in essence is not such, although it is "created by immobile, constant in time surface charges". And further, we have an almost directly opposite statement:

"Thus, a constant current in a conductor cannot be maintained by Coulomb forces alone" [25, p. 116].

Nikolaev [31] emphasised that "...a stationary electric field significantly differs from an electrostatic one" [31, p. 98]. But at the same time, further on – they (the electrostatic field and EPPCC) have many common properties. "Both are potential..., which indicates the non-closedness of their lines: these lines begin and end on charges or at infinity" [31, p. 100].

But the fashion in explaining this issue was set in the well-known manual by D. V. Sivukhin:

"Thus, in the case of stationary currents, macroscopic electric charges can be only on the surface or in places of inhomogeneity of the conductive medium. In this respect, the electric field of stationary currents is analogous to the electrostatic one. The analogy between these fields goes even further.

If the currents are stationary, then the density of electric charges at each point in space does not change in time, *although there is a movement of electricity: new ones continuously come to the place of departing electric charges. Such charges, as shown by experiment (as well as Maxwell's equations), create the same Coulomb field in the surrounding space as immobile charges of the same density. From this, it follows that the electric field of stationary currents is a potential field.*

The electrostatic field is a Coulomb field of immobile charges. Inside conductors with equilibrium of charges, it is equal to zero. *The electric field of stationary currents is also a Coulomb field, but the charges that excite it are in motion. Therefore, the field of stationary currents exists inside conductors*" [39, p. 38].

But in reality, neither experiments nor Maxwell's equations show or can show that moving charges create the same Coulomb field as immobile charges of the same density.

The electric field of moving charged particles fundamentally differs from the field of immobile CP [22]. First, the electric field of MCP is not potential. Second, neglecting even tiny relativistic corrections in the expressions for the vectors **E** and **B** of the electromagnetic field of MCP in the analysis of electromagnetic phenomena leads to the "loss" of the physical phenomenon in theoretical cognition [22] (see more details in [24]).

Therefore, the statement that moving charges create the same Coulomb field in the surrounding space as immobile charges of the same density, is erroneous.

Let's compare further. In the textbook [25], it is stated that SEFCC is created by immobile charges, and in [39], on the contrary, the idea is put forward that this field is created by moving charges.

In the monograph by G. A. Ryazanov [37], it is considered (contrary to the statement that "lines of intensity begin and end on charges, or at infinity" [31, p. 100]), that the force lines of SEFCC inside the conductor, through which current flows, do not come out of the charges that create this field, although they are its "sources", but pass by them: "The force lines of the stationary electric field *inside* a conductor carrying a current *do not come out of the charges that create this field and which are its 'sources', but pass by them*" [37, p. 40].

"Let us emphasise that the charges covering the surface of a conductor through which a constant current flows arise as a result of the accumulation of charged particles *that participate* in the process of charge transfer and continuously *change* one another. However, their movement does not change the distribution of charges on the surface of the conductor. This distribution is continuously renewed in the process of current flow" [37, p. 40].

That is, in the manual [39], it is stated that SEFCC *does not differ from the Coulomb field of immobile electrons of the same density*, and in [37], on the contrary, it is stated that this *field "differs slightly" from the Coulomb field, but, although these surface charges are the "source" of SEFCC, the force lines of such a field do not start on these charges, but pass by them.*

If this field differs even slightly from the Coulomb field, then it cannot be potential in principle. Indeed, when explaining similar phenomena in all manuals, a model of a conductor with current is used, in which positive ions are immobile, and conduction electrons move with drift velocity. Since the electron moves, the intensity of its electric field is determined by formula (1). And a field described by formula (1) is non-potential.

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{l} \times \mathbf{r}}{r^3} \tag{1}$$

Then how can the total electric field of the set of conduction electrons be potential if the field of each electron separately is clearly non-potential?

Of course, we consider that the principle of superposition holds.

In the manual by I. E. Irodov, the theses of the manual by D. V. Sivukhin [39] are repeated.

"If the currents are stationary, then the distribution of electric charges in the conductive medium does not change in time, although there is a movement of charges: in each point, new ones continuously come to the place of departing charges. *These* *moving charges create the same Coulomb field as immobile charges of the same configuration. Therefore, the electric field of stationary currents is a potential field...The electric field of stationary currents is also a Coulomb field, but the charges that excite it are in motion. Therefore, the field of* **E** *in stationary currents exists inside conductors with current*" [11, p. 124].

Let's now highlight the main conclusions of the given statements.

1. In the educational literature, there are directly opposite statements regarding the state of motion of those surface charges that create EPPCC. In most manuals, it is persistently repeated that "there is a movement of charges: in each point, new ones continuously come to the place of departing charges" [11, 37, 39].

Coming from the specifics of the phenomenon of electric current flow, it is necessary to consider, in our opinion, that it is these moving charges that create the electric field in a conductor with current.

That is, despite the fact that the distribution of surface charges remains unchanged in time, this distribution is caused by moving CP. However, this stationarity is dynamic—stationarity as a consequence of the movement of CP.

At the same time, in some manuals, it is considered that SEFCC inside the conductor is created by "*immobile, constant in time surface charges*" [25].

2. It is considered that EPPCC is Coulomb, although it is created by moving charges [11, 39]. Therefore, EPPCC is potential. Of course, the problem does not arise when it is considered that the charges are immobile [25].

These two positions are incompatible with each other. Indeed, the electric field of a CP that moves uniformly and linearly is non-potential. Then it remains unclear how the set of moving CP can create a potential electric field if the field of each separate conduction electron is clearly non-potential. In our view, this contradiction is fundamental in explaining the properties of EPPCC in the manuals cited above.

Moreover, the authors of the cited manuals *do not pay attention to the nature of the surface charges* that create, for example, in a cylindrical CCC and in its vicinity, a stationary electric field of constant current.

3. Relativistic nature of the stationary electric field of a conductor with current

Indeed, one should agree that the charges located on the surface of a conductor through which current flows arise as a result of the accumulation of charged particles that participate in the process of charge transfer and which continuously change one another. The movement of charges does not change the average value of the macroscopic surface charge density on the surface of the CCC.

And since the electric field of MCP is non-potential, then how to reconcile this position with the fact that SEFCC is potential in this case.

It is the absence of a clear answer to this question in a number of manuals on electrodynamics [11, 25, 31, 37, 39] that forces the assertion that moving CP create the same Coulomb field as immobile ones, that is, EPPCC is a potential field.

That is, in existing manuals, this contradiction is resolved simply: although the CP that create EPPCC are moving, their field remains Coulomb, potential. And there is no problem.

On the other hand, if we consider the electric field of the CCC to be non-potential, then the very possibility of, for example, measuring voltage in a constant current circuit becomes unclear. And from practice, it is known that SEFCC is potential, and therefore, based on measuring the potential difference at the ends of a conductor of length l, we find the tangential component of the vector **E** on the surface of the CCC.

Based on the principle of superposition, one should expect that the electric fields generated by, for example, a charged filament moving uniformly along its length, and a linear procession of charged particles (model of a conductor with constant current), are non-potential.

Therefore, let us estimate the magnitude of the non-potentiality of the electric field of a linear procession of CP, and to what extent (in the case of linear conductors with constant current) this non-potentiality of the electric field can manifest itself or be measured experimentally. Solving this issue will make it possible to substantiate and explain the well-known experimental fact of practical electrical engineering—determining the intensity of SEFCC inside the conductor itself by measuring the voltage drop between two points of a conductor with constant current.

In addition, the general conclusions obtained in our works [22, 23] (see [24]), for methodical purposes, need to be illustrated with a simple and transparent example that can be used as a model of the corresponding physical phenomenon.

Using the method described in section 2.3 and in section 3.3 of the dissertation [23], we find the electric field intensity at an arbitrary point in space *P*, which is created by a uniformly charged filament (procession of charged particles) moving with a constant velocity v along its length (figure 1).

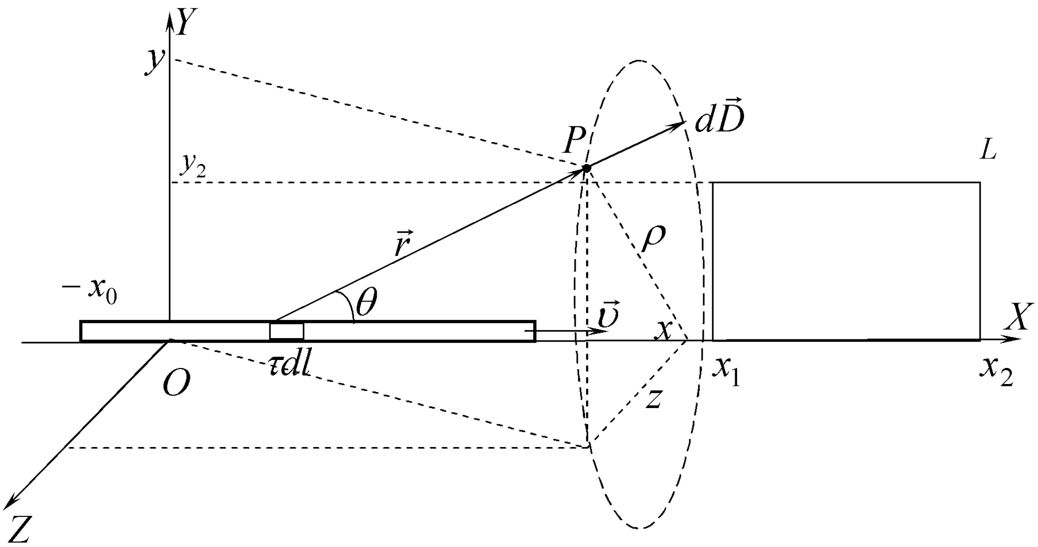

Figure 1: Uniformly charged thread moving with constant velocity v along its length. τ is the linear charge density.

In the case when the EMF of a linear conductor with constant current is considered, τ is the linear charge density of conduction electrons in the laboratory FR.

That is, $\tau = \frac{q \cdot n_-^0 \cdot S}{\sqrt{1-\beta^2}}$, where n_-^0 is the concentration of conduction electrons in the proper FR, q is the magnitude of the electron charge, S is the cross-sectional area of the CCC.

When finding the components of the electric field D_x, D_y, D_z at an arbitrary moment of time, it should be taken into account that the limits of integration depend on the initial position of the filament. If we have chosen the coordinate system as shown in figure 1, then the integration limits at time t are from $x_0 + vt$ to $l - x_0 + vt$.

For example, the position of the linear procession of CP shown in figure 2 corresponds to the integration limits from $x_0 + vt$ to $l - x0 + vt$.

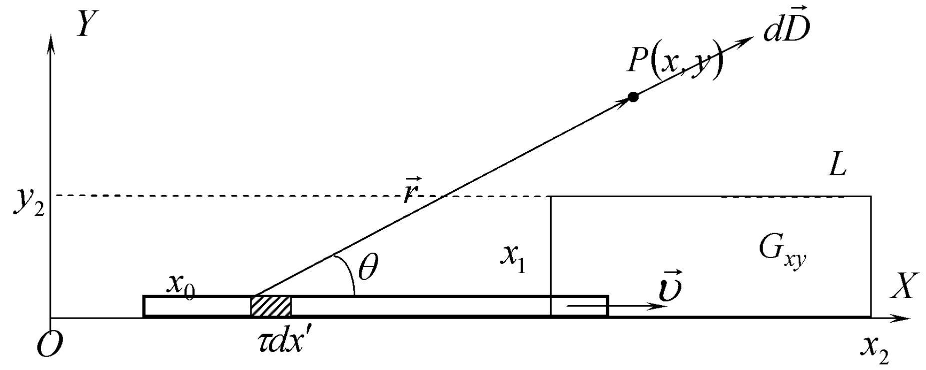

Figure 2: EMF of a moving charged thread considered in the XOY plane.

From now on, we will denote the limits as a and b, with always $b-a=l=l_0\sqrt{1-\beta^2}$ when talking about the field of a moving filament. But when talking about the model of CCC, obviously, $b-a=l_0$.

After uncomplicated but painstaking calculations for $\nabla\times\mathbf{D}$ we have:

$$\nabla\times\mathbf{D}=\mathbf{j}\frac{\tau\cdot\beta^2 z\cdot(1-\beta^2)}{4\pi}K(x,y,z,t)-\mathbf{k}\frac{\tau\beta^2 y(1-\beta^2)}{4\pi}\cdot K(x,y,z,t), \tag{2}$$

where

$$K(x,y,z,t)=\left(\frac{1}{((x-b)^2+\rho^2(1-\beta^2))^{\frac{3}{2}}}-\frac{1}{((x-a)^2+\rho^2(1-\beta^2))^{\frac{3}{2}}}\right). \tag{3}$$

Thus, the electric field at any point in space outside the current segment of length l (moving charged filament of length $l=l_0\sqrt{1-\beta^2}$), is vortex ($\nabla\times\mathbf{D}\neq 0$).

At the same time, with a symmetric orientation of the contour l relative to the filament $x_1-a=-(x_2-b)$ and $x_2-a=-(x_1-b)$, the circulation of the vector $\mathbf{E}$ equals zero: $\oint\mathbf{E}\cdot d\mathbf{l}=0$.

During the analysis of the EMF of a separate moving CP ([16], [24, p. 71-81]), it was obtained that the total EMF in any contour, which is caused both by the non-potentiality of the electric field of the MCP and by the phenomenon of electromagnetic induction, equals zero. Based on the PR principle of superposition, a similar situation should be expected in our case as well.

That is, it is necessary to compare the values $\nabla\times_z\mathbf{D}=-\frac{\tau\cdot\beta^2\cdot y\cdot(1-\beta^2)}{4\pi}\cdot K(x,y,z,t)$ and $-\varepsilon_0\frac{\partial B_z}{\partial t}$.

The induction of the magnetic field generated by the moving charged filament equals [22, 23]:

$$B_z=\frac{\mu_0\cdot i}{4\pi\cdot y}\cdot\left(\frac{x-a}{\sqrt{(x-a)^2+y^2(1-\beta^2)}}-\frac{x-b}{\sqrt{(x-b)^2+y^2(1-\beta^2)}}\right).$$

If at the initial moment the left end of the rod was at the origin of our FR, then the integration limits are as follows: $a=vt, b=l+vt$, where $l=l_0\sqrt{1-\beta^2}$.

$$B_z(t)=\frac{\mu_0\cdot i}{4\pi\cdot y}\cdot\left(\frac{x-vt}{\sqrt{(x-vt)^2+y^2(1-\beta^2)}}-\frac{x-(l+vt)}{\sqrt{(x-(l+vt))^2+y^2(1-\beta^2)}}\right). \tag{4}$$

Then the value $\frac{\partial B_z}{\partial t}$ equals:

$$\frac{\partial B_z}{\partial t}=\frac{\mu_0\tau v^2 y(1-\beta^2)}{4\pi}\cdot K(x,y,z,t).$$

Thus, we have equality:

$$(\nabla \times \mathbf{D})_z = -\frac{\tau \cdot \beta^2 \cdot y \cdot (1-\beta^2)}{4\pi} \cdot K(x,y,z,t) = -\varepsilon_0 \frac{\partial B_z}{\partial t}. \tag{5}$$

That is, at any point in space at any moment of time in the vicinity of CCC or a moving charged filament, the non-potential electric field is ***compensated*** by the vortex electric field caused by the change in the induction of the magnetic field in time, $\frac{\partial \mathbf{B}}{\partial t}$. Here $\mathbf{B}(x,y,z,t)$ is the induction of the magnetic field generated by the linear procession of CP (segment of a conductor with constant current).

Thus, one of the features of the mechanism of creation and existence of potential SEFCC becomes clear.

In any case, it is clear why when a constant current flows through a homogeneous conductor, its electric field is potential and thanks to what measuring voltage on a section of a constant current circuit makes it possible to determine the intensity of SEFCC inside the conductor.

That is, the process of current flow, the procedure of measuring voltage and current, explanation of physical phenomena in this process cannot be explained without contradictions and understood without field representations and the principle of relativity.

If relativistic corrections in the expression for the electric field intensity of MCP and the procession of charged particles are not taken into account, then, possibly, the potential nature of SEFCC becomes clear. The phrase “possibly, becomes clear” means that the explanation of this phenomenon in various scientific and methodical publications reflects inconsistency and inconsistency in the interpretation of the nature of SEFCC both within the framework of a separate manual and when comparing this interpretation in different literary sources. But then, we face serious contradictions with PR. In particular, a contour made of metal wire would heat up if it were in the field of CCC.

If relativistic corrections are taken into account and **only exact expressions for the electric field intensity** $\mathbf{E}$ **of MCP** or the procession of particles are used, then we come to the conclusion that there will be a non-potential electric field in the space around the CCC. This non-potential electric field of CCC, although of insignificant magnitude, in principle could be measured and its manifestations observed.

If, at the same time, the exact, relativistic **expression for the induction** of the magnetic field is also taken into account, then there is a **compensation** of the non-potential electric field. Such explanations and ideas create and form a holistic and non-contradictory picture of the mechanism of electromagnetic processes in a constant current circuit.

Again, we are convinced that only theory can explain what is measured or observed in physical experiments.

The result of the analysis of these models (figures 1 and 1) also refutes contradictions in the interpretation of imaginary experiments shown in figures 3-6 of [24, p. 48-50].

4. Volume charge of a conductor with constant current and the condition of neutrality of a conductor with current

In most educational manuals on electromagnetism [6, 11, 26, 39, 41, 43], it is stated that in a motionless conductor with current, the volume charge density equals zero. Indeed, this follows from the law of charge conservation for constant currents

$$\nabla \cdot \mathbf{j} = 0 \tag{6}$$

and Coulomb’s law in the form:

$$\nabla \cdot \mathbf{D} = \rho. \tag{7}$$

From equation (6), we get

$$\nabla \cdot \mathbf{j} = \nabla \cdot (\lambda \cdot \mathbf{E}) = \nabla \cdot \left(\frac{\lambda \cdot \mathbf{D}}{\varepsilon \cdot \varepsilon_0} \right) = 0, \tag{8}$$

where $\mathbf{j}$ is the current density, $\mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}$, $\mathbf{E}$ is the electric field intensity inside the conductor with constant current, λ is the specific electrical conductivity of the material from which the conductor is made.

Thus, $\nabla \cdot \mathbf{D} = 0$, and therefore the volume charge density of the CCC equals zero:

$$\rho = 0. \tag{9}$$

This conclusion is valid under the following conditions:

- λ = const;
- if the action of the conductor's own magnetic field on the conduction electrons (pinch effect) is neglected [42];
- if the dependence of the volume charge density on the velocity of movement of a certain charge distribution is neglected [22].

It should be noted that the electric field both inside and outside the CCC can also be caused by the pinch effect.

Indeed, if we take into account the action of the Lorentz force from the magnetic field of the current on the conduction electrons (pinch effect), then from the law of charge conservation $\nabla \cdot \mathbf{j} = 0$ and Maxwell's equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$ we get (since $\mathbf{j} = \lambda(\mathbf{E} + \mathbf{v} \times \mathbf{B})$,

$$\nabla \cdot \mathbf{D} = \nabla \cdot (\varepsilon \cdot \varepsilon_0 \mathbf{E}) = \varepsilon\varepsilon_0 \mathbf{v} \cdot \nabla \times \mathbf{B} = \frac{\varepsilon \cdot \mathbf{vj}}{c^2} = \rho. \tag{10}$$

Thus, a conductor with current is characterised by:

a) a volume charge density (ϵ = 1) [15, 26]

$$\rho = \frac{\varepsilon \cdot \mathbf{vj}}{c^2} = \rho_- \frac{v^2}{c^2}, \tag{11}$$

where $\rho_- = \frac{\rho_-^0}{\sqrt{1-\beta^2}}$ is the volume charge density of conduction electrons in the reference frame K in which the conductor is motionless; ρ_-^0 is the volume charge density of conduction electrons in the proper FR;

b) a surface charge density [42, p. 322]

$$\sigma = \frac{r_0 j \mathbf{v}}{2c^2} - \frac{j\left(\frac{\pi r_0^2 \mathbf{v}}{c^2} - \frac{z}{\lambda}\right)}{4\pi \cdot r_0 \ln \frac{r_0}{a}}. \tag{12}$$

But there is yet another physical phenomenon which, in the generally accepted model of CCC, can lead to the emergence of an additional electric field.

This additional electric field is caused by the difference in the values of charge densities of the set of electrons and ions of the crystal lattice due to their movement with different velocities in an arbitrary FR [6, 15, 20, 44].

For the clarity of our further argumentation, let's briefly recall the relativistic interpretation of the interaction of a moving charged particle and CCC ([6], [1, p. 338], [19, p. 151-156], [20, p. 278-280]).

Let in the reference frame K along the OX axis there be a motionless CCC. Along it, an electron moves with velocity $\mathbf{v}$. Find the force acting on the electron in the FR K and in the FR K'. $\mathbf{V}$ is the velocity of the FR K' relative to the FR K (figure 3).

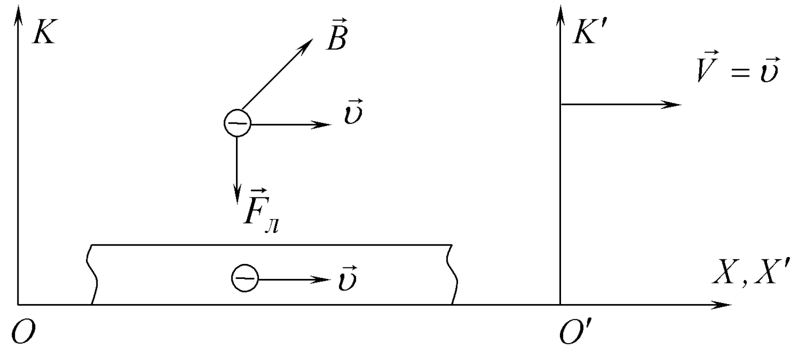

Figure 3: Interaction between an electron and a conductor with direct current in reference frames K and K'.

Deliberately considering a simplified problem, when $V = v$, with the aim of most transparently showing the contradictory nature of views regarding the condition of neutrality of CCC.

The electron in FR K' is motionless, therefore a force can act on it only from some electric field. This field in FR K' is created by uncompensated densities of charges of ions and conduction electrons. Since electrons in FR K' are motionless, and ions move with velocity $V = v = v'$, their charge density respectively equals

$$\rho'_{-} = \rho^0_{-} = \rho_{-}\sqrt{1-\beta^2} \tag{13}$$

$$\rho'_{+} = \frac{\rho^0_{+}}{\sqrt{1-\beta^2}}, \tag{14}$$

where $\beta = \frac{v}{c}$, c is speed of light in vacuum; ρ^0_{-}, ρ^0_{+} are *densities of charges of conduction electrons and positive ions, respectively, in their own reference frames.*

And therefore the volume charge density of the CCC in FR K' will be as follows

$$\rho' = \rho'_{+} + \rho'_{-} = \frac{\rho^0_{+}}{\sqrt{1-\beta^2}} - \rho_{-}\sqrt{1-\beta^2} = \frac{\rho^0_{+}\beta^2}{\sqrt{1-\beta^2}} \tag{15}$$

Here, the condition of neutrality of a motionless CCC [6, 27, 43, 44] was used

$$\rho^0_{+} = -\frac{\rho^0_{-}}{\sqrt{1-\beta^2}} = -\rho_{-}. \tag{16}$$

After finding the electric field intensity created by the volume charge (15), for the force acting on a motionless electron in FR K' we get $F'_y = \frac{F_y}{\sqrt{1-\beta^2}}$, which is what the principle of relativity requires.

Of course, the drift velocity of conduction electrons is extremely small compared to the speed of light. But, as we have made sure, neglecting even tiny relativistic effects when analysing electromagnetic phenomena leads to the "loss" of the physical phenomenon in theoretical cognition [16, 17, 22] (see also [24, p. 71-81]).

Perhaps for the first time, attention was drawn to the contradiction of the neutrality condition (16) in the work of G. V. Nikolaev [31]: since FR K and FR K' are equivalent, then when $\mathbf{V} = \mathbf{v}$ due to symmetric conditions that determine the movement of electrons and protons respectively in FR K and in FR K', the physical situations in these FRs should be the same.

In scientific and methodical literature, three ways of resolving this contradiction have been proposed:

Method 1. "...the physical properties of negative and positive charges turn out to be different" [31]. But, if the electron and proton in electromagnetic interactions manifest themselves symmetrically, then "we are forced to recognise the existence of physical non-equivalence of the laboratory FR, motionless on the surface of the massive body Earth that creates a gravitational field, in relation to any other FR that moves relative to it" [31, p. 6].

Method 2. A conductor with current is neutral in that FR which moves with the drift velocity of conduction electrons [26, p. 92], that is, in FR K':

$$\rho' = \rho'_+ + \rho'_- = 0, \tag{17}$$

where $\rho'_+ = \frac{\rho^0_+}{\sqrt{1-\beta^2}}$, $\rho'_- = \rho^0_- = \rho_- \sqrt{1-\beta^2}$.

The argumentation of the authors of the article [26] is as follows. Let's assume the appearance of a volume charge of a conductor with current in FR K' (15) and consider the interaction of conduction electrons of the CCC with the field of this volume charge. In FR K, their movement causes a current density $\rho_-\mathbf{v}$. Due to the fact that in FR K' "the magnetic field does not act on the conduction electrons and there is no other force that could balance the action of the electric field of the volume charge" [26, p. 91], it is necessary to require the fulfilment of (17).

Let's make several critical remarks regarding this thesis:

A. In reality, from the point of view of FR K', there is nothing to balance, because, according to, for example, such a transformation formula for the projection of the intensity $\mathbf{E}$ of the electromagnetic field $E'_y = \frac{E_y - VB_z}{\sqrt{1-\frac{V^2}{c^2}}}$ [19, 21] when transitioning from FR K to FR K', $E'_y = 0$ when $\mathbf{V} = \mathbf{v}$.

In the reference frame K, the electric field intensity of the volume charge $\rho = \rho_- \frac{v^2}{c^2}$, which in [26] is considered to be caused (mistakenly) by relativistic effects, $E_y = E_r$ is compensated by the field of the Lorentz force $E^L_r = v \cdot B_z$. In FR K', the purely electric field, which in FR K was equal to E_y, increases and becomes equal to $\frac{E_y}{\sqrt{1-\frac{V^2}{c^2}}}$, and the "moving" magnetic field B_z creates in FR K' an electric field of the same magnitude, but opposite direction $-\frac{v \cdot B_z}{\sqrt{1-\frac{V^2}{c^2}}}$. Thus, in FR K' everything is again in balance (as required by the principle of relativity), and there is no need to "balance the action of the electric field of the volume charge". And therefore, condition (17) is incorrect.

B. From the neutrality condition (17), it follows that in FR K, the volume of the motionless CCC is charged. Then the density of this charge, taking into account (13), (14), and (17), equals:

$$\rho = \rho_+ + \rho_- = \rho^0_+ + \frac{\rho'_-}{\sqrt{1-\beta^2}} = \rho^0_+ + \frac{\rho^0_-}{\sqrt{1-\beta^2}} = \rho_- \cdot \frac{v^2}{c^2}. \tag{18}$$

The charge density (18) exactly equals the charge density that appears during the pinch effect (see formula (11)) in a motionless CCC [15, 26]. But the pinch effect and the increase in volume charge density during the movement of any set of charged particles are different physical phenomena. And the authors [26] probably wanted to explain the pinch effect by relativistic effects.

C. In FR K, an external electron moving with drift velocity along the conductor, besides the Lorentz force, will also be acted upon by an electric force from the surface charge. This positive surface charge is formed due to the movement of some electrons inside the conductor. "It is this surface charge in FR K' that will attract the external motionless electron" [26, p. 92].

But in FR K' both $F_y^L = 0$ (because the external electron is motionless in FR K'), and $\rho' = 0$, which means the surface charge is also zero. An obvious violation of the principle of relativity.

D. A clear violation of physical relativity is observed in the solution proposed in [26].

Let us recall that, according to V. A. Fock, one should distinguish physical relativity, as a principle, "which affirms the existence of corresponding phenomena, and the simple fulfilment of the requirement of covariance of equations during the transition from one reference system to another" [7, p. 245]. Indeed, let's imagine in FR K an electron moving along the CCC with the drift velocity of conduction electrons and a motionless proton.

Then in FR K' we have a symmetric picture in kinematic terms. But an analysis of the forces acting on the electron and proton in both FRs shows the physical non-equivalence of situations in FR K and in FR K' (see section **??**).

Method 3. The condition of neutrality of a conductor proposed by us [13]:

$$\rho_+^0 = -\rho_-^0. \tag{19}$$

Then in FR K, a motionless conductor with current is characterised by a volume charge density [22, 23]:

$$\rho = \rho_+^0 - \frac{\rho_-^0}{\sqrt{1-\beta^2}} = \frac{\rho_0}{\sqrt{1-\beta^2}}\left(\sqrt{1-\beta^2} - 1\right). \tag{20}$$

Let us provide reasoning in favour of conditions (19) and (20):

A. Indeed, if the condition (16), $\rho_+^0 = -\rho_-$, is true during current flow, then this means that $\rho_-^0 < \frac{\rho_-^0}{\sqrt{1-\beta^2}} = \rho_-$, that is, the motionless, as a whole, set of conduction electrons has a lower charge density than the charge density of this same set of electrons, but moving.

Then with $\beta = 0$ (no current in the conductor) $\rho_-^0 = \rho_- < \rho_+^0$, that is, a conductor without current (after the current is turned off) will be charged positively: $\rho_+^0 - \rho_-^0 = \rho_+^0\left(1 - \sqrt{1-\beta^2}\right)$, and the magnitude of this charge depends on the strength of the current that flowed earlier [22]?!

B. The density of any charge distribution when transitioning from one FR to another is transformed according to the formula $\rho = \frac{\rho^0}{\sqrt{1-\frac{V^2}{c^2}}}$ [20, p. 275], where V is the velocity of movement of some charge distribution with density ρ_0, ρ is the charge density in the FR relative to which this charge distribution moves.

Therefore, the charge densities of the set of electrons and ions when moving with an arbitrary, but the same in magnitude, velocity should increase by the same factor. If a conductor without current is neutral $\rho_+^0 = -\rho_-^0$ (which is natural), then why after current appears and the set of electrons moving at velocity v is described, as is known, by a charge density $\rho_- = \frac{\rho_-^0}{\sqrt{1-\frac{v^2}{c^2}}}$, this density ρ_- does not become numerically greater than ρ_+^0?

C. If the densities of the set of electrons and ions when moving with the same velocity change in the same way, then a conductor without current that is moving (under the condition of neutrality (16)) will be charged with a volume density [22]:

$$\rho = \rho_+ - \rho_- = \frac{\rho_+^0}{\sqrt{1-\beta^2}}\left(1 - \sqrt{1-\beta^2}\right).$$

But there are no physical grounds to consider a motionless (as well as one moving with constant velocity) conductor without current as charged.

D. The current density in FR K equals $j_k = \rho_- v$. And in FR K' (figure 3), the current is caused by the movement of only ions with the same magnitude of velocity as the movement of conduction electrons in FR K.

But the current density in FR K' is greater $j'_x = \frac{\rho_+^0 v}{\sqrt{1-\beta^2}} > j_x$, despite the fact that the charge densities of both electrons and ions increase in the same way in the FR relative to which they move with equal velocities.

Similarly, we will find that in FR K' the magnetic field is greater than in FR K: $B'_z = \frac{B_z}{\sqrt{1-\beta^2}}$, but the physical situations are identical (up to the sign of moving and motionless charged particles).

E. Based on the neutrality condition of a conductor with current (16) we get:

- in FR K the electric field is absent, and in FR K' the electric field intensity equals $E'_y = \frac{\rho_- S\beta^2}{2\pi\varepsilon_0 a\sqrt{1-\beta^2}}$;
- the induction of the magnetic field in FR K $B_z = \frac{\mu_0 \rho_- v S}{2\pi a}$, and in FR K' the induction of the magnetic field is $\frac{1}{\sqrt{1-\beta^2}}$ times greater, because $I' = \frac{I}{\sqrt{1-\beta^2}}$ despite the fact that ions in the K'-system move with the same velocity (in magnitude) as the movement of conduction electrons in the K-system.

F. The violation of physical relativity when transitioning from FR K to FR K' is particularly clearly visible if we compare the force acting on an electron in FR K (its velocity v) and on a positron that moves in FR K' with velocity $v' = v$ along the current.

According to the traditional view, the magnitude of the force acting on the electron equals $B_z q\vec{v}$, while the force acting on the positron in RF K' equals $|q\vec{E}'_y - q\vec{v}' \times \vec{B}'_z| = 0$, even though the physical conditions in systems K and K' are identical [22]. *In this example, the possibility of physical adaptation appears to be violated, which "proves to be a decisive condition for the realization of physical relativity"* [8, p. 8-9].

Note that in [33] an attempt was made to analyze a similar problem. However, in our opinion, it contains some incorrect conclusions and also repeats our results from many years ago.

In connection with this, for the expressions of volume charge density and volume current density in RF K, justified using transformation formulas for 4-current components and based on the physical meaning of these quantities, in the general case when $V \neq v$ and within each neutrality condition for conductors with constant current (16) and (19), we obtain a consistent solution [22].

Indeed, under the neutrality condition (16) we have:

$$j'_x = \Gamma j_x, \tag{21}$$

$$\rho' = \frac{V j_x}{c^2}\Gamma, \tag{22}$$

where $\Gamma = \frac{1}{\sqrt{1-B^2}}$, $B = \frac{V}{c}$, $j_x = \rho_- v$, $\rho_- = \frac{\rho_-^0}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$.

Now, if we assume that the conductor with constant current in its proper reference frame is characterized by volume charge density (20) and $\rho_+^0 = |\rho_-^0| = \rho_0$, then the current density and charge density in RF K' according to the transformation formulas for 4-current components and based on the physical meaning of these quantities, equals:

$$j'_x = \rho_0(v' - V)(1 + \gamma\beta^2) \tag{23}$$

$$\rho' = \rho_0\gamma\beta^2(1 - \beta^2) \tag{24}$$

Returning to the discussion of the results in [33], where it is claimed that with the compensation of the electric field of moving conduction electrons by the electric field of stationary ions, analysis of the interaction between a stationary proton in RF K and a conductor with constant current leads to a contradiction with the principle of relativity, because with such compensation the interaction force between this proton and the conductor equals zero. But if we move to another inertial reference frame, even with this compensation, the resultant force acting on the external proton will no longer be zero.

The latter is incorrect.

If in RF K $F = 0$, then in any other RF the resultant force also equals zero. Indeed, in RF K' this proton will be affected by both the Lorentz force and the force from the electric field caused by the volume charge (19) (in RF K there is complete field compensation, $\rho = 0$).

Then, taking into account (21) and (22), the resultant force acting on the proton in an arbitrary inertial RF K' equals (figure 4)

$$F'_p = F'_L - F'_E = qVB_z{}' - qE'_y = 0.$$

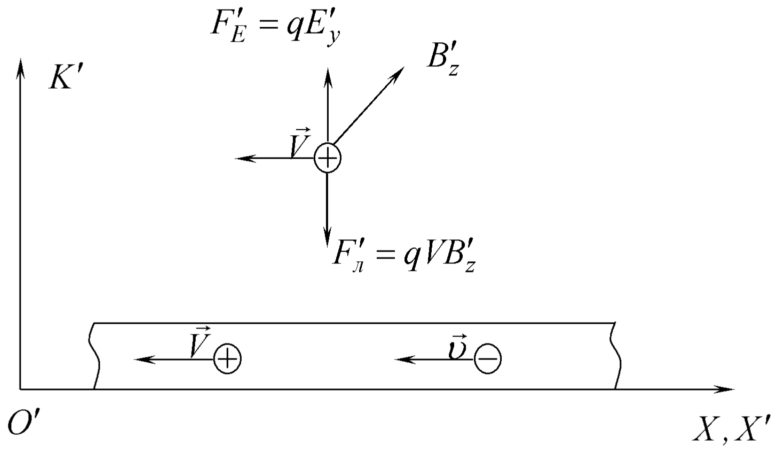

Figure 4: Forces acting on a proton in reference frame K'.

If we assume that compensation is absent (as we proposed earlier [13]), and a force F acts on the proton in RF K from a uniformly charged conductor with current with volume charge density (22)

$$F = E_y q = \frac{q\rho_0 S\gamma(\sqrt{1-\beta^2}-1)}{2\pi\varepsilon_0 d},$$

then in any other RF, as can be verified using (23) and (24), $F'_p = F$. Moreover, the educational problem analyzed in [33] was solved by us much earlier in [13].

If we accept conditions (19), (20), then all contradictions outlined in points 3A-3E are resolved.

If condition (20) corresponds to physical reality, then, in our opinion, one should not literally understand the words “charge of a conductor with current”. The additional electric field of a conductor with constant current is the result of increasing the transverse component of the electric field intensity of moving charged particles. Then we can say that the field caused by volume density (20) is a superposition of the field of a linear collection of stationary ions and the field of a linear collection of moving conduction electrons [22].

Perhaps condition (20) has no relation to reality due to the crudeness of the conductor with constant current model. Because we assume that the electron and ion subsystems in electromagnetic phenomena related to current flow do not interact with each other in any way, and exist as two independent linear chains of charges. But then it is necessary to explain why for arbitrary values of β for any values of current flowing through the conductor, the strange neutrality condition $\rho_+^0 = -\frac{\rho_-^0}{\sqrt{1-\beta^2}}$ is realized, if in the absence of current $\rho_+^0 + \rho_-^0 = 0$?

And in the methodology of teaching physics, such a model is actively used. Possibly, these aporia are generated not only by the imperfection of the model, but primarily by the dialectic of the cognition process itself. But teaching physics, in our opinion, should be such that students see these contradictions, understand the reasons for their appearance, and look for ways to overcome them.

Therefore, when studying this issue, one should clearly formulate the problem (which phenomenon is being analyzed, which model of conductor with current is being considered) and not use non-relativistic approximations. One should show the contradictions that arise in the analysis of both the field of a conductor with current and the interaction of a charged particle moving relative to the conductor.

Thus, let's summarize this analysis.

When a constant or quasi-stationary current passes through a metallic cylindrical conductor in its own reference frame, a stationary electric field of constant current arises, caused by surface charges.

Taking into account the pinch effect, the total surface charge density is determined [42, p. 322] by formula (12).

When the pinch effect and relativistic changes in volume charge densities are not taken into account, then, as already noted, in the proper reference frame of the conductor with current, the volume charge of the conductor with constant current $\rho = 0$ (9).

The pinch effect in the classical model of a conductor with constant current, which we are discussing, causes the appearance of volume charge density (11).

Taking into account relativistic effects and under the neutrality condition (16), a conductor with current in its own reference frame is neutral.

But under the neutrality condition proposed by us (19), in the proper reference frame, a conductor with current should be considered "charged" with volume charge density (20).

5. Relativistic description of the interaction between two long conductors with currents

Regarding the electric field caused exclusively by relativistic effects in the change of charge densities of conduction electrons and ions of the crystal lattice, there are no direct experimental confirmations of this phenomenon yet. But the analysis of the CCC model, which is widely used in the methodology of teaching physics, based on consistent and systematic use of STR methods, inevitably, irreversibly leads to the conclusion that in its own FR, a conductor with constant current is characterised by a volume charge density (20):

$$\rho = \rho_+^0 - \frac{\rho_-^0}{\sqrt{1-\beta^2}} = \frac{\rho_0}{\sqrt{1-\beta^2}}\left(\sqrt{1-\beta^2} - 1\right),$$

where $\rho_0 = \rho_+^0 = -\rho_-^0$ is the volume charge density of conduction electrons and ions of the crystal lattice in their own reference frames; $\beta = \frac{v}{c}$; v is drift velocity of conduction electrons in the CCC.

The existence of volume and surface charges on the CCC leads to additional components of electromagnetic interaction between conductors with currents.

For example, taking into account SEFCC in the scheme in figure 5 leads to the fact that between two segments of CCC, the electric force of attraction between them can be greater than the magnetic force of repulsion [11, p. 172].

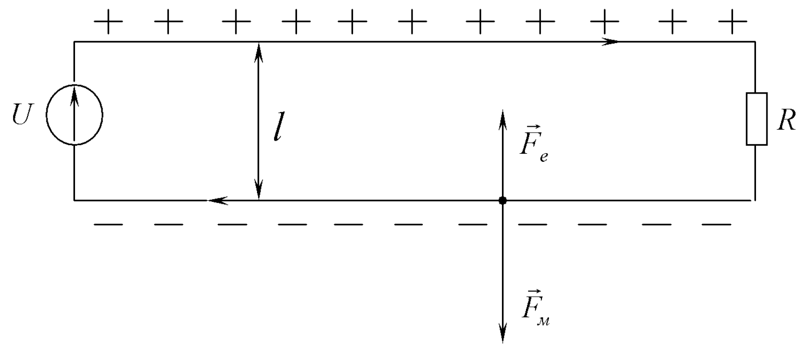

Figure 5: Electromagnetic interaction of two parallel conductors with currents [11].

And the resultant force of interaction between parallel CCCs will be equal to zero if $\frac{F_m}{F_e} = 1$.

This condition will be fulfilled when $R = R_0 = \frac{\ln \eta}{\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}} = 360$ Ohm, where $\eta = \frac{l}{r}$, l is a distance between the wires, r is a radius of the cross-section of the wire, and R is an active resistance of the load.

If $R < R_0$, then $F_m > F_e$ – the conductors repel each other.

If $R > R_0$, then $F_m < F_e$ – the conductors attract each other.

Thus, the statement that conductors through which currents of opposite direction flow repel each other is true only when the electrical part of the interaction can be neglected, that is, with a sufficiently small resistance R in the circuit shown in figure 5.

"In addition, having measured the force of interaction between conductors with current (and force is always measured as a resultant), we cannot, generally speaking, determine the current strength I. This must be kept in mind to avoid misunderstandings" [11, p. 173].

In connection with the reform of the system of school and higher education, the revision of the content of physics education, in connection with the growing role of physical models and model experiments in the process of teaching physics [21], the exact and consistent solution of electrodynamic models, which are traditionally used in educational literature, acquires important significance for the didactics of physics and the formation of the physical style of thinking of students.

In previous sections, it is shown that the exact consideration of relativistic corrections, despite their insignificant magnitude in real electrodynamic phenomena, in the description of the interaction of moving charged particles is fundamentally important and necessary.

In educational and scientific-methodical literature [6, 28, 36, 43], to illustrate the fundamental property of the electromagnetic field – the relativity of electric and magnetic fields, a popular example is the following.

The magnetic interaction of a moving charged particle with other moving charged particles (with a linear conductor through which current flows) is represented as a purely electric interaction due to relativistic changes in the electric fields of moving CP (see [20, 21, 36] and section 4).

This example initiates an attempt at a similar interpretation of the interaction of two infinitely long conductors with currents [21, p. 219].

That is, considering that the CCC in its own FR is characterised by a volume charge density (24), let's find the force of interaction between two such parallel CCCs, taking *into account only the magnetic component and the electric component caused by the relativistic effect (24).*

But first, let's describe the interaction between two conductors with constant currents, models of which are unnaturally idealized in some electrodynamics textbooks.

In this case, we consider the following model of conductors with currents ([28, 36], [38, p. 121-125]): in each conductor, there is an equal number of positive and negative charge carriers that move with the same speeds in opposite directions (figure 6).

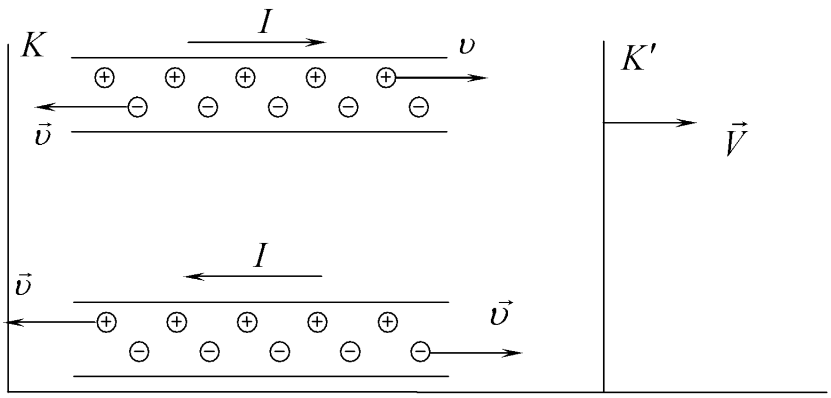

Figure 6: Models of two conductors with direct current, in which positive ions and electrons have velocities of the same magnitude but opposite directions.

To calculate the force acting on a unit length of one of these conductors from the other in the laboratory reference frame (LRF) K, it is necessary to calculate the force acting on the positive charge carriers in their own reference frame and on the negatively charged particles in their own reference frame. Then the values of these forces should be recalculated according to the transformation formulas for force components [21, 22, 43] in RF K.

Indeed, let the velocities of the charges in RF K equal $\mathbf{v}$, the linear charge density in the proper reference frame $|\tau_0^+| = |\tau_0^-| = \tau_0$. The distance between the conductors with constant currents equals a.

It is clear that the forces acting on the positive and negative charge carriers of the first conductor in their own reference frames will have an electrical nature. The electric field in the proper reference frame of the positive or negative charge carriers of the first conductor is caused by the different values of the linear density of positive and negative charges of the second conductor.

As a result, we get [24, appendix B]

$$F_y = F'_y\sqrt{1-\beta^2} = (F'_+ - F'_-)\sqrt{1-\beta^2} = \frac{\mu_0 I^2}{2\pi a}\Delta l, \tag{25}$$

where the current $I = \frac{2\tau_0 v}{\sqrt{1-\beta^2}}$.

However, in our opinion, a more adequate model of a conductor with current is depicted in figure 7.

Here, the positive charge carriers are stationary (which corresponds to stationary ions of the crystal lattice of the conductor), and only conduction electrons move with drift velocity $\mathbf{v}$ relative to the LRF K.

Let's assume that the conductor without current is neutral $|\tau_+^0| = |\tau_-^0| = \tau_0$; this corresponds to the case when the chain of negative charge carriers is stationary relative to the positively charged chain of ions.

Problem: Find the interaction force between two conductors with constant currents, models of which are depicted in figure 7. In the proper reference frame, the conductors

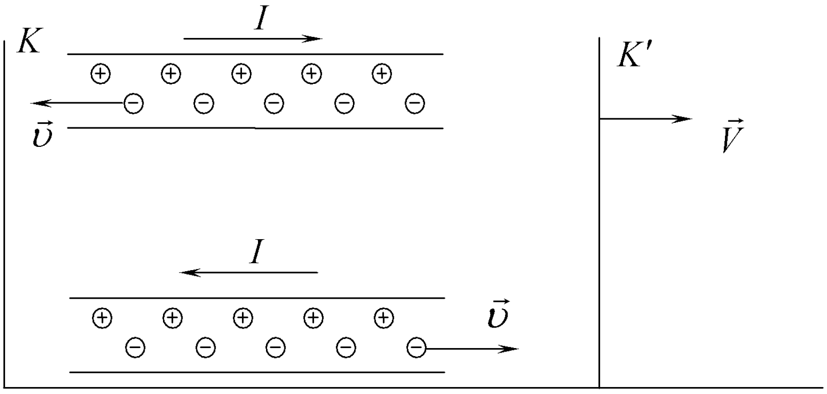

Figure 7: Realistic model of conductor interactions with direct current. It is assumed they are “charged” in their own reference frame with volume charge density (20).

with currents are characterized by volume charge density (20). The distance between the conductors with constant currents equals a.

Solution:

Method 1. To find the interaction force between two currents shown in figure 7, we will implement the program of the 1st method described in [24, appendix B]. Let’s move to the reference frame (RF K') connected with the conduction electrons of the first conductor. Then the electrons, which are stationary in this RF, experience a force from the electric field, caused by the unequal Lorentz contractions of the electron and ion chains of the second conductor (this will be a repulsive force):

$$F'_e = E_2 \cdot \tau_0 \Delta l' = \frac{\tau_2}{2\pi\varepsilon_0 a} \cdot \tau_0 \Delta l' = \frac{\tau_0^2}{2\pi\varepsilon_0 \cdot a \cdot \sqrt{1-\beta^2}} \left(\frac{1+\beta^2}{\sqrt{1-\beta^2}} - 1 \right) \cdot \Delta l',$$

where $\tau_2 = \tau_2^+ - \tau_2^- = \frac{\tau_0}{\sqrt{1-(\beta_2)^2}} - \frac{\tau_0}{\sqrt{1-\beta^2}} = \frac{\tau_0(1+\beta^2)}{(1-\beta^2)} - \frac{\tau_0}{\sqrt{1-\beta^2}}$ is the resultant linear charge density of the second conductor in the reference frame of the conduction electrons of the first conductor; $\beta_2 = \frac{v}{c} = \frac{1}{c} \cdot \frac{2v}{1+\beta^2}$.

And the stationary ions of the first conductor in RF K will experience an attractive force equal to

$$F_{+e} = \tau_0 \Delta l \cdot E_- = \tau_0 \Delta l \frac{\tau_0}{2\pi\varepsilon_0 a} \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right).$$

F'_- is the force acting on the electrons of the first conductor with constant current, which are **stationary** in RF K'. When recalculating its value for RF K, as perpendicular to the velocity of motion, and taking into account that $\Delta l' = \frac{\Delta l}{\sqrt{1-\beta^2}}$, we get

$$F_{-e} = F'_{-e}\sqrt{1-\beta^2} = \frac{\tau_0^2}{2\pi\varepsilon_0 \cdot a \cdot \sqrt{1-\beta^2}} \left(\frac{1+\beta^2}{\sqrt{1-\beta^2}} - 1 \right) \cdot \frac{\Delta l}{\sqrt{1-\beta^2}} \sqrt{1-\beta^2}.$$

Thus, for the total repulsive force between two conductors, which is per unit length $\Delta l = 1$ m of one of them in RF K, we have

$$F_y = F'_{-e}\sqrt{1-\beta^2} - F_{+e} = \frac{\mu_0 I^2}{2\pi a} \left\{ \frac{2}{\beta^2} \left(1 - \sqrt{1-\beta^2} \right) \right\}, \tag{26}$$

where the current now equals $I = \frac{\tau_0 v}{\sqrt{1-\beta^2}}$.

This last result can also be obtained using the method proposed by us in [12, 14].

Method 2. In reality, the electron distribution of the first conductor is subject to the following forces from the electron and ion distributions of the second conductor: electric force $F^e_{-2,-1}$ and magnetic force $F^m_{-2,-1}$ from the electron chain, and electric force $F^e_{+2,-1}$ from the stationary ion chain.

The stationary ions of the first conductor are subject to forces $F^e_{-2,-1}$ and $F^e_{+2,+1}$ respectively in the electric fields created by the moving electrons and stationary ions of the second conductor.

Therefore, the resultant interaction force per unit length of one of them equals

$$F_y = F^m_{-2,-1} + F^e_{-2,-1} - F^e_{+2,-1} - F^e_{-2,+1} + F^e_{+2,+1} = \frac{\mu_0\tau_0 v}{2\pi a\sqrt{1-\beta^2}} \cdot \frac{\tau_0 v}{\sqrt{1-\beta^2}} + \\ + \frac{\tau_0}{2\pi\varepsilon_0 a\sqrt{1-\beta^2}} \cdot \frac{\tau_0}{\sqrt{1-\beta^2}} - \frac{\tau_0}{2\pi\varepsilon_0 a} \cdot \frac{\tau_0}{\sqrt{1-\beta^2}} - \frac{\tau_0}{2\pi\varepsilon_0 a\sqrt{1-\beta^2}} \cdot \tau_0 + \\ + \frac{\tau_0}{2\pi\varepsilon_0 a}\tau_0 = \frac{\mu_0 I^2}{2\pi a}\left\{\frac{2}{\beta^2}\left(1-\sqrt{1-\beta^2}\right)\right\},$$

which, naturally, coincides with (26).

Method 3. An even simpler method for finding the force of interaction between "charged" conductors with currents is to calculate the magnetic and electric components of this interaction.

Indeed,

$$F^m = I \cdot \Delta l \cdot B_2 = \frac{\tau_0 v}{\sqrt{1-\beta^2}} \cdot \Delta l \cdot \frac{\mu_0\tau_0 v}{2\pi a\sqrt{1-\beta^2}} = \frac{\tau_0^2\beta^2}{(1-\beta^2)2\pi\varepsilon_0 a},$$

$$F^e = \frac{\tau_0}{2\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) \cdot \tau_0\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) \cdot \Delta l,$$

$$F_y = F^m + F^e = \frac{\mu_0 I^2}{2\pi a} \cdot \left\{\frac{2}{\beta^2}\left(1-\sqrt{1-\beta^2}\right)\right\}.$$

Thus, finding the interaction force between two conductors with constant currents, models of which are depicted in figure 7, gives the result:

$$F_y = \frac{\mu_0 I^2}{2\pi a} \cdot \left\{\frac{2}{\beta^2}\left(1-\sqrt{1-\beta^2}\right)\right\}. \quad (27)$$

Each of these methods illustrates different manifestations of electromagnetic interaction and different descriptions of EM interaction in a specific example.

This illustrates the principle of relativity, the methodology of its use, shows the invariance of the magnitude of the force of this interaction, confirms the credibility of the obtained result, and its difference from the classical formula.

If we consider the interaction of conductors with currents within the framework of the adopted model for equally directed currents (figure 8), then the curly brackets in expression (27) for the interaction force, as it turns out, equal [12, 14]:

$$\left\{2\left(1+\frac{\sqrt{1-\beta^2}-1}{\beta^2}\right)\right\}.$$

That is, the interaction force between parallel conductors with constant currents, which is per unit length of one of them, with equally directed currents equals

$$F_y = \frac{\mu_0 I^2}{2\pi a} \cdot \left\{2\left(1+\frac{\sqrt{1-\beta^2}-1}{\beta^2}\right)\right\}. \quad (28)$$

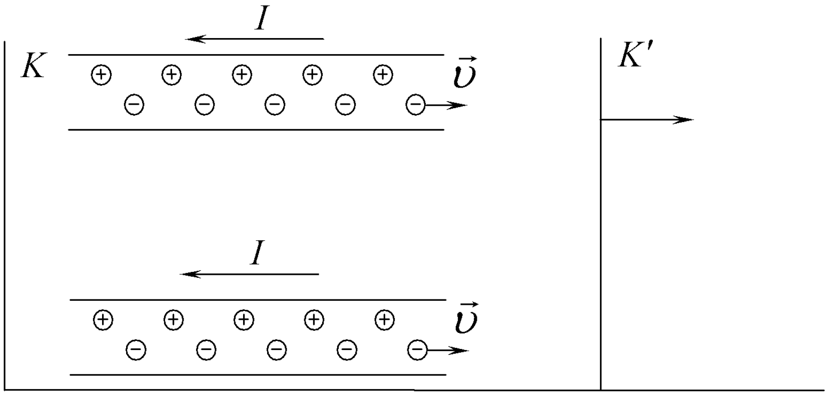

Figure 8: Describing the interaction of two conductors with direct current in a model where they are "charged" in their own reference frame [12, 21] (parallel currents).

Thus, interpreting the interaction of two conductors with currents, the use of models (figures 7, 8) which is traditional and natural in the teaching methodologies of electromagnetism in higher and secondary schools, based on a consistent relativistic approach and based on the general didactic principles of fundamentality, consistency, and systematicity, we obtained an expression for the interaction force that differs from the generally accepted formulas (27) and (28).

For the values of the correction coefficients

$$\left\{\frac{2}{\beta^2}\left(1-\sqrt{1-\beta^2}\right)\right\} \tag{29}$$

$$\left\{2\left(1+\frac{\sqrt{1-\beta^2}-1}{\beta^2}\right)\right\} \tag{30}$$

in formulas (27) and (28) for antiparallel and parallel currents respectively when $\beta << 1$ (which is obviously the case in practical electrical engineering), expanding in a series $\sqrt{1-\beta^2} \approx 1-\frac{\beta^2}{2}$ with high accuracy we get 1.

The graphs of the dependence of the correction coefficients (29) and (30) on the drift velocity of conduction electrons in the CCC are given in figure 9.

It is clear from the preceding (section 4) that the appearance of coefficients (29) and (30) in the formulas for the force (27) and (28) is due to taking into account relativistic corrections in the electric interaction of moving electrons.

This additional interaction, which is responsible for the multipliers (29) and (30), can also be explained as the interaction of "charged" conductors with currents, since within the framework of the accepted model and assumptions, the electric field intensity of a conductor through which current flows, equals

$$E = E_- - E_+ = E_0\left(\frac{1}{\sqrt{1-\beta^2}}-1\right), \tag{31}$$

where $E_0 = \frac{\tau_0}{2\pi\varepsilon_0 \cdot a}$.

This field can be viewed as created by a negatively "charged" thin conductor with a resultant linear charge density

$$\tau = \tau_- - \tau_+ = \frac{\tau_0}{\sqrt{1-\beta^2}} - \tau_0.$$

It seems that the words "charged conductor with current" should not be understood literally, but any charged particle that is outside such a conductor should be acted

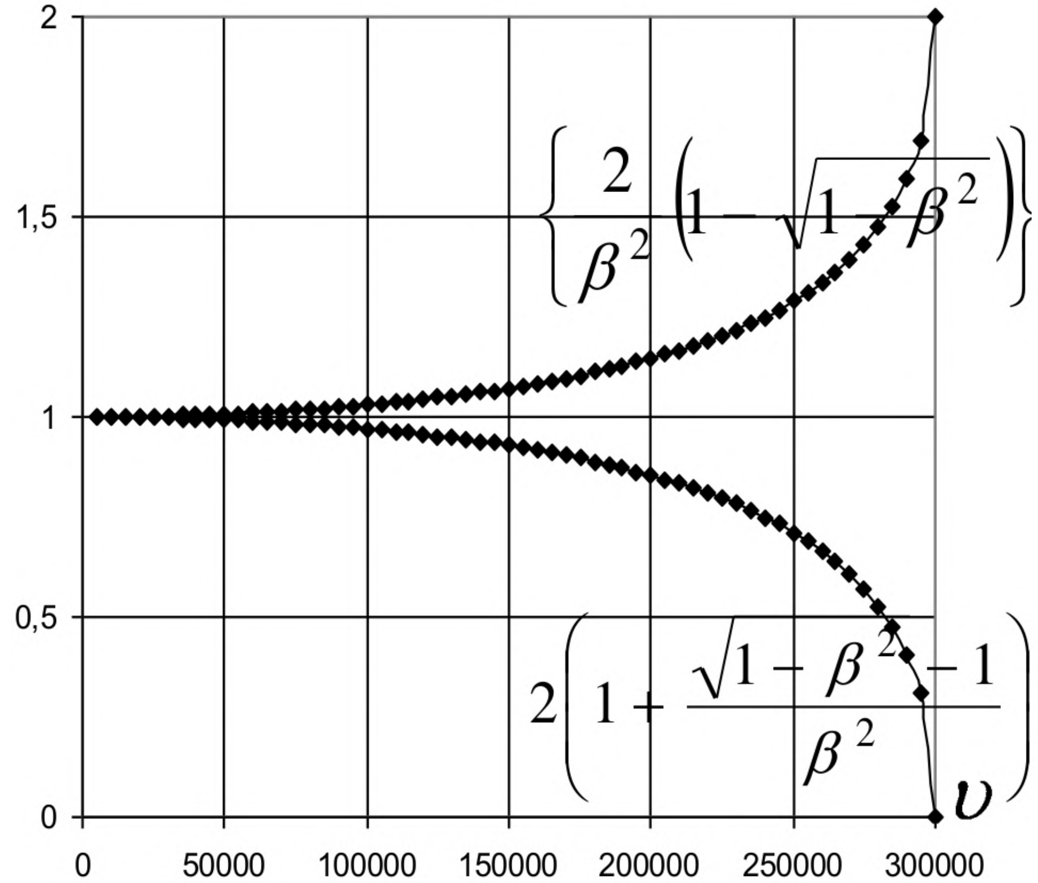

Figure 9: Graphs of functions $f_{\uparrow\downarrow}(v) = \left\{\frac{2}{\beta^2}\left(1-\sqrt{1-\beta^2}\right)\right\}$, $f_{\uparrow\uparrow}(v) = 2\left(1+\frac{\sqrt{1-\beta^2}-1}{\beta^2}\right)$.

upon by a force from the field whose intensity is given by expression (31). We believe that due to the movement of the chain of electrons, the electric field outside the conductor with current is caused by an increase in the transverse direction of the electric field intensity of the moving chain of electrons by $\frac{1}{\sqrt{1-\beta^2}}$ times. And in this case, there is no need to talk about the "charge of a conductor with current". After all, when considering the interaction of 2 charged particles in an FR, relative to which they move in a direction perpendicular to the line connecting them, the force of electrical interaction between them increases by $\frac{1}{\sqrt{1-\beta^2}}$ times, but it is not claimed that the increase in the electric force is due to an increase in the magnitude of the charge by $\frac{1}{\sqrt{1-\beta^2}}$ times as a result of its movement.

Of course, the CCC model, which was discussed above, is far from the real physical situation.

For example, the assumptions about how and why the forces acting on the electronic and ionic subsystems add vectorially remain completely unsubstantiated (and incomprehensible); why the interaction of these subsystems within the framework of the accepted model is not taken into account within each of the conductors, etc.

Possibly, the coefficients (29) and (30), are caused precisely by the inadequacy of the accepted model of the conductor with current to the objective reality.

In many educational manuals on electrodynamics [6, 10, 28, 36, 41–43], models similar to those shown in figures 6, 7, 8 are used.

With their help, fundamental properties of the electromagnetic field and many important laws of electromagnetism are substantiated; these models are actively exploited by didactics. Therefore, their analysis should be carried out, if possible, to the end and consistently; but then with inevitability one should accept the results (27), (28), (31).

At the same time, it is obvious that the main features of electromagnetic phenomena are transmitted quite well by such a model (figure 7, 8).

Further, this idealised problem (figure 6) in a number of manuals is used to show the relativistic nature of the magnetic field ([38, p. 122-125], [36, p. 172-178]).

But in these manuals, an *extremely idealised model of the CCC* is used (figure 6), in which positive and negative charge carriers move with the same velocities in opposite directions.

And, possibly, in the excellent manuals [36, 38], such a model of a conductor with current (figure 6) was deliberately chosen, which, due to its symmetry, makes it possible to avoid considering the issue of "charge of a conductor with current".

Relativistic-invariant description of the interaction between conductors with currents. Recently, articles have been appearing in the domestic methodical periodicals, the content of which largely repeats the results of our research from 20 years ago [12, 14]. In addition, they contain errors in the calculations of the models being discussed. This was emphasised by us in the work [15]. Therefore, it makes sense to return to these issues and discuss them in more detail.

It was shown [12, 14] (section 4) that the relativistic analysis (within the framework of classical electron theory, as is usually accepted in manuals on the basics of electrodynamics [6, 25, 36, 38, 39, 41]) of the interaction of 2 infinitely long straight conductors with currents, models of which are shown in figures 7 and 8, leads to the following expressions for the force, which falls on a unit length of conductors, respectively, for parallel and antiparallel currents:

$$F_y = \frac{\mu_0 I^2}{2\pi \cdot a} \cdot \left\{ 2\left(1 + \frac{\sqrt{1-\beta^2}-1}{\beta^2}\right) \right\} \tag{32}$$

$$F_y = \frac{\mu_0 I^2}{2\pi \cdot a} \cdot \left\{ \frac{2}{\beta^2}\left(1 - \sqrt{1-\beta^2}\right) \right\}. \tag{33}$$

In the last expressions, v is the drift velocity of conduction electrons in FR K, a is the distance between conductors with constant current (CCC), c is the speed of light in vacuum, $I = \frac{\tau_0 v}{\sqrt{1-\beta^2}}$ is the magnitude of the current strength, $\tau_0 = \rho_-^0 S$ is the linear charge density, ρ_-^0 is the volume charge density of conduction electrons in the proper reference frame, S is the cross-sectional area of the linear metal conductor, $\beta = \frac{v}{c}$.

Current strength is a quantity that is a combination of other quantities and through which it is convenient, at the electrotechnical level of understanding electromagnetic phenomena, to describe both the interaction of chains of moving charged particles, and the magnetic field that accompanies such processions of CP. Simply convenient and nothing more.

We have shown that the MF of such processions of CP is generated not by conduction currents, but by displacement currents [24, p. 53-62], and the induction of the MF can still be expressed through a certain combination of quantities that characterise the movement of CP (through the so-called current strength), and which is measured directly in experiments. Therefore, such attention in the methodology is paid to the concept of "current strength", although this quantity does not carry a significant physical meaning.

Obviously, the multipliers in the curly brackets of formulas (32) and (33) for any values of current strengths that are actually encountered in practical electrical engineering, differ very little from unity. For example, if current of $\sim$120 A flows through two copper conductors with a diameter of 2 mm, then the drift velocity of electrons in each conductor is $\approx$ 0.3 cm/s. If the distance between the currents a = 3 cm, then, expanding $\sqrt{1-\beta^2}$ in a power series, for the interaction force we get

$$F = 10\left\{1 \pm \frac{\beta^2}{4}\right\}\frac{\text{din}}{\text{cm}} \tag{34}$$

with an accuracy up to quantities of the second order of smallness in β.

The sign "–" in (34) corresponds to equally directed currents, and "+" to oppositely directed currents. The difference of (27) and (28) from the classical expression $F_y = \frac{\mu_0 I^2}{2\pi\cdot a}$ is so insignificant and minuscule that a direct experiment on measuring the force of interaction of currents, is not able, at the modern level of experimental technology [14, 40], to register the difference between the forces (27) and (**??** from the value determined by the classical expression.

The most important argument in substantiating the truth or falsehood of (27) and (28) is the results of the experiment. However, as far as is known, no such special experiments have been performed, and the entire set of facts is completely described by the classical formula $F_y = \frac{\mu_0 I^2}{2\pi\cdot a}$.

At the same time, since the correction multiplier $\left\{1 \pm \frac{\beta^2}{4}\right\} \approx \left\{1 \pm 3\cdot 10^{-23}\right\}$ differs from unity by a minuscule amount, and since the sensitivity of the available experimental technology [14, 40] is insufficient for the refutation or confirmation of formulas (27) and (28), the electrotechnical confirmation of the expression $F_y = \frac{\mu_0 I^2}{2\pi\cdot a}$ cannot be regarded as proof of the erroneousness of (27) and (28).

After all, even when considering the forces of interaction between real conductors with currents, the classical electric force caused by the appearance of excess surface charges on the surface of these conductors [11, 29] is not taken into account. This is usually not discussed, because the ratio of this electrical force of interaction to the purely magnetic force of interaction of parallel currents is of the order of $\frac{F_e}{F_m} \approx 10^{-12}$ [29].

Among theoretical arguments in favour of the correctness of some relations, it is worth pointing to the requirement of their relativistic invariance [7, 32, 44]. The principle of relativity leads to completely defined laws of transformation of physical quantities during the transition from one inertial reference frame to another.

Let us show for the situation depicted in figure 8 that the expression (28), for example, satisfies the formulas of transformation for force [1, 21] when transitioning from one reference frame to another.

To find the force of interaction of currents in FR K', which moves relative to the laboratory reference frame K along the OX axis with velocity $\mathbf{V}$, it is necessary to know the volume current density and volume charge density by which the conductors are characterised in FR K'. Obviously, these quantities are defined [13, 15] by expressions (23) and (24).

Then the electric field intensity and the induction of the magnetic field at each point of the second conductor in FR K', respectively, equal

$$E' = \frac{\tau'}{2\pi\varepsilon_0\cdot a} = \tau_0\frac{\sqrt{1-\beta^2}-1+B\beta}{\sqrt{1-B^2(1-\beta^2)}}\cdot\frac{1}{2\pi\varepsilon_0\cdot a}, \tag{35}$$

$$B'_z = \frac{\mu_0 I'}{2\pi\cdot a} = \frac{\tau_0}{2\pi\varepsilon_0 c^2\cdot a}\cdot\frac{v+V\sqrt{1-\beta^2}-V}{\sqrt{(1-B^2)(1-\beta^2)}}, \tag{36}$$

where $\tau_0 = \rho_0 S$ is the linear charge density of a motionless chain of charged particles, $|\rho_+^0| = |\rho_-^0| = \rho_0$, v is the velocity of movement of electrons of conductivity in FR K, S is the area of the cross-section of the conductor, $\beta = \frac{v}{c}$, $B = \frac{V}{c}$, $v' = \frac{V-v}{1-B\beta}$ is the velocity of movement of conduction electrons in FR K'.

Fields E' and B'_z can also be found using the formulas of transformation of components of the electromagnetic field tensor [13, 18].

The force of interaction between currents in FR K', which falls on the length $\Delta l\sqrt{1-B^2}$ and equals $F'_y = F'_A - F'_E$, where F'_A is the Ampere force, acting on the current I' from the field B'_z; F'_E is the force of electrical interaction between currents.

That is, in FR K' we measure the force that falls on the same segment as in FR K (Δl = 1 cm), but in FR the length of it is equal to $\Delta l\sqrt{1-B^2} = \sqrt{1-B^2}$; if we take a segment of length $\Delta l' = 1$ cm according to the measurements of FR K', then we get $F'_y = F_y$.

Thus, taking into account (35) and (36), we get ([12], [21, p. 223]):

$$F'_y = \sqrt{1-B^2}\left(\frac{\mu_0 I'^2}{2a\pi} - \frac{\tau'^2}{2\pi\varepsilon_0 \cdot a}\right) = \frac{\mu_0 I^2}{2\pi \cdot a} \cdot \left\{2\left(1 + \frac{\sqrt{1-\beta^2}-1}{\beta^2}\right)\right\} \cdot \sqrt{1-B^2} \qquad (37)$$

in full accordance with the formulas of transformation of components of 4-force.

However, we will find the same relation between the forces acting between conductors in systems K and K' ($F'_y = F_y\sqrt{1-B^2}$) and within the framework of the traditional approach, that is, considering that conductors with current are “charged” only in that reference frame, relative to which they move.

Description of the interaction of two parallel CCCs in the traditional model. Indeed, in the traditional model, it is considered, as has already been noted, that in the proper FR, a conductor with current is neutral, “uncharged”, and the neutrality condition $\rho_+^0 = -\frac{\rho_-^0}{\sqrt{1-\beta^2}} = -\rho_-$.

The force of magnetic interaction in the proper FR between two parallel CCCs that falls on a unit length of one of them in such a model is determined by the classical formula $F_y = \frac{\mu_0 I^2}{2\pi \cdot a}$.

Let us find the force of interaction between them in FR K'.

In the system K', relative to which currents move along their length with velocity V, besides the magnetic interaction of currents, there will also be an electric interaction, because each conductor with current in this FR is characterised by a volume charge density [21, p. 236] (see also (22))

$$\rho' = \frac{\rho_+^0}{\sqrt{1-B^2}} - \frac{\rho_-^0}{\sqrt{1-\beta'^2}} = \frac{V \cdot v \cdot \rho_-^0}{c^2\sqrt{(1-B^2)(1-\beta^2)}} = \frac{V \cdot j_x}{c^2\sqrt{1-B^2}},$$

where $j_x = \frac{\rho_-^0 \cdot v}{\sqrt{1-\beta^2}}$ is the current density in FR K, relative to which the conductor with current is motionless, v is the drift velocity of conduction electrons in this FR; $\beta = \frac{v}{c}$, $B = \frac{V}{c}$, $v' = \frac{V-v}{1-B\beta}$ is the drift velocity of conduction electrons in FR K'.

The condition of neutrality of a conductor with current in the proper FR is determined by relation (16). Therefore, the resultant force acting on a section of the conductor of length $\Delta l \cdot \sqrt{1-B^2}$ of one of the currents from the other equals:

$$F' = \frac{\mu_0 I'}{2a\pi} \cdot I'\Delta l\sqrt{1-B^2} - \frac{\tau'}{2a\pi \cdot \varepsilon_0} \cdot \tau'\Delta l\sqrt{1-B^2} =$$

$$= \Delta l\sqrt{1-B^2} \cdot \left(\frac{\mu_0 I'^2}{2a\pi} - \frac{\tau'^2}{2a\pi \cdot \varepsilon_0}\right) = \frac{\mu_0 I^2 \Delta l \cdot \sqrt{1-B^2}}{2a\pi},$$

where $I' = \left(\frac{\rho_+^0 \cdot V}{\sqrt{1-B^2}} - \frac{\rho_-^0 \cdot v'}{\sqrt{1-\beta'^2}}\right) S = \frac{S \cdot v \cdot \rho_-^0}{\sqrt{(1-B^2)(1-\beta^2)}} = \frac{I}{\sqrt{1-B^2}}$; $\tau' = \rho' S = \frac{VI}{\sqrt{1-B^2} \cdot c^2}$, respectively, the current strength in the conductor and the linear charge density on this conductor in FR K'.

That is, we got the necessary relation between the forces F' and F:

$$F' = F\sqrt{1 - \frac{V^2}{c^2}}.$$

Thus, to obtain a relativistic-invariant description of the interaction of currents within the framework of the traditional approach, it is necessary to have the following law of current transformation: $I' = \frac{I}{\sqrt{1-B^2}}$, where I is the magnitude of the current in the reference frame K.

Note that the relation $I' = \frac{I}{\sqrt{1-B^2}}$ is obtained immediately from the formulas of transformation of components of 4-current [20, p. 275] under the condition $s_4 = 0$, or proceeding from the definition of current strength (see also section 4 and expression (21)). Such duality in solving this problem within the framework of the same approximation (the same degree of roughness of the accepted models) causes surprise. But do we have in both cases (in FR K and in FR K') to do with physical relativity in the context of the statement of V. A. Fock [7, p. 245].

For this, let us consider the electromagnetic field created by a conductor with current in two reference frames: the system K (laboratory) and the FR associated with the conduction electrons – K_e. In the system K_e, within the framework of the solution proposed by us, we have a situation identical (up to the sign of moving and motionless CP) to such in the system K. Indeed, in FR K, positive ions are motionless, and conduction electrons move with velocity v. In FR K_e, electrons are motionless, and ions move with velocity v. Therefore, the electric field intensity E and the induction of the magnetic field B have in the system K the same magnitude as in the system K_e due to the symmetry of the conditions in systems K and K_e.

Within the framework of the traditional solution [6, 27, 43, 44] in the system K, the electric field is absent, because the neutrality condition of the conductor with current (16) is fulfilled

$$|\rho_-| = \left|\rho_-^0(1-\beta^2)^{-\frac{1}{2}}\right| = |\rho_+^0|.$$

And in the system K_e, on the contrary, the electric field is not equal to zero:

$$E_e = \frac{\rho_+^0 S \cdot v^2}{c^2 2\pi \cdot a\varepsilon_0\sqrt{1-\beta^2}}, \tag{38}$$

where a is the distance of the point of the field from the procession of charged particles (from the CCC).

The induction of the magnetic field in the system K equals

$$B = \frac{\mu_0 I}{2\pi \cdot a} = \frac{\mu_0}{2\pi \cdot a} \cdot \frac{\rho_-^0 \cdot v \cdot S}{\sqrt{1-\beta^2}},$$

and in the system K_e, the induction of the magnetic field:

$$B_e = \frac{\mu_0 \cdot \rho_-^0 v \cdot S}{2a\pi(1-\beta^2)},$$

because the current in the system K_e turns out to be equal to $I_e = \frac{I}{\sqrt{1-\beta^2}}$ (see above, or [20, p. 275]) despite the fact that this current is caused only by the movement of ions with the same magnitude of velocity v as the velocity of electrons in the system K.

These conclusions can be illustrated using figure 10.

The systems K and K_e are symmetric (identical) up to the sign of CP that move in these reference frames. The obvious non-equivalence of reference frames K and K_e in the traditional model, which is widely used in the study of electrodynamics, is evident.

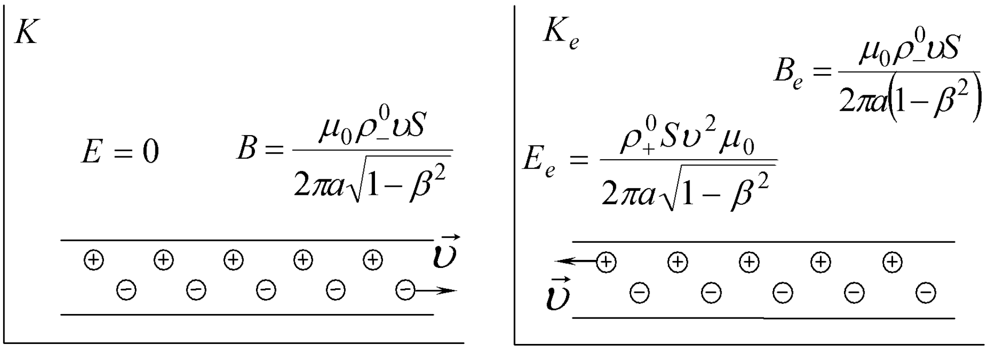

Figure 10: EMF in reference frames K and K_e in the traditional model of a conductor with direct current.

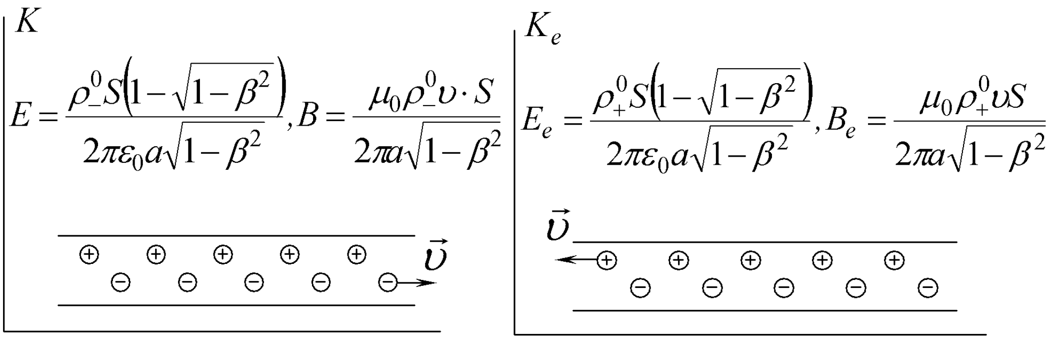

Figure 11: EMF in reference frames K and K_e in the model of a conductor with direct current that we analysed in [12, 13, 15].

With the use of the neutrality condition (19), we have the same characteristics of the EMF both in FR K and in FR K_e (figure 11).

The violation of physical relativity is especially clearly seen when comparing the force acting on an electron that moves in the system K in the direction of movement of conduction electrons with velocity v, equal to the velocity of the latter, on the one hand, and the force acting on a positron in the system K_e, moving in this reference frame (K_e) with velocity v in the direction of movement of ions.

We have absolutely identical situations both in FR K and in FR K_e. The traditional point of view gives for the force acting in FR K on a charged particle with charge q:

$$F = qvB = qv\frac{\mu_0 \rho_-^0 vS}{2\pi a\sqrt{1-\beta^2}}. \tag{39}$$

And in the system K_e, the force acting on a positron equals:

$$F = qvB_e - qE_e = qv \cdot \frac{\mu_0 \rho_-^0 vS}{2a\pi(1-\beta^2)} - q\frac{\rho_+^0 S \cdot v^2}{c^2 2\pi a \cdot \varepsilon_0\sqrt{1-\beta^2}} = 0, \tag{40}$$

despite the fact that the conditions in systems K and K_e are the same (figure 12). In this example, it seems, the possibility of physical adaptation is violated, which "is a decisive condition for the realisation of physical relativity" [8, p. 8, 9].

If we adopt the model of a conductor with current proposed by us [12, 13, 15], then the solution of this problem in reference frames K and K_e will be identical (figure 13) [15, 21].

Thus, discussing the relativistic description of the interaction of two CCCs, for the force acting on a unit length of one of the currents, we obtain, as has already been noted, a consistent solution within the framework of each of the models.

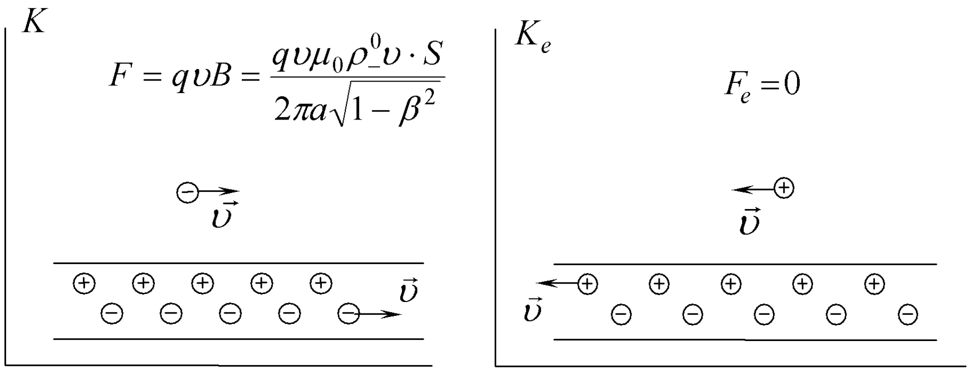

Figure 12: The force acting on an electron in the EMF in reference frame K and on a positron in reference frame K_e in the traditional model of a conductor with direct current.

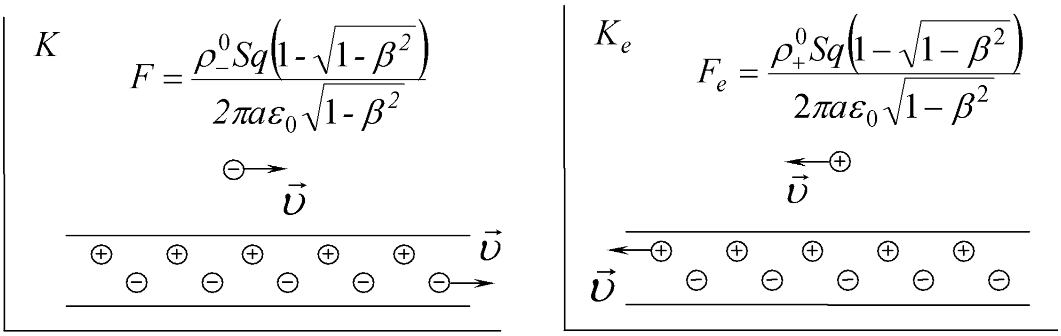

Figure 13: The force acting in the electric field on an electron in reference frame K and a positron in reference frame K_e in the model of a conductor with direct current that we discussed in [12, 13, 15].

But based on the principles of symmetry and relativity, it should be recognised that the description of the interaction of two CCCs, the models of which are so intensively used in the didactics of electromagnetism and which is proposed by us, to the maximum extent satisfies the considerations of simplicity and a certain beauty.

It should be noted that when describing the interaction of the proposed models of 2 CCCs, we did not take into account other physical causes of the appearance of additional interactions, besides the one being discussed, and which is caused by the flattening of the electric field of a moving CP. Namely: the electrical interaction between the CCCs, which arises due to the appearance of surface charges on the CCC surface, and the electrical interaction caused by the pinch effect in the CCC.

6. Conclusion

1. An analysis of educational and methodological literature on explaining the nature and properties of stationary electric field of direct current (SEPDC) and the state of motion of those surface charges that generate SEPDC was conducted. It was revealed that contradictions and inconsistencies in interpreting the nature of SEPDC relate to the state of motion of surface charges and the potentiality or non-potentiality of the electric field of direct current.
2. From first principles, expressions for the electric field strength generated by a linear procession of charged particles and a uniformly moving charged thread were derived, and *it was shown that these fields are non-potential.*
3. The equivalence of the electromagnetic field of a charged thread moving uniformly and a conductor with direct current of the same length, where the drift

velocity of conduction electrons equals the velocity of the thread's motion, was demonstrated.

4. Based on the principle of relativity, an explanation for the experimental non-observability of the non-potential electric field of a linear procession of charged particles was proposed, thus resolving contradictions in interpreting the properties of SEPDC.
5. It was shown that when analysing any electrodynamic phenomena, the use of non-relativistic approximations (formulas, expressions, relations) is incorrect. Neglecting even minute relativistic effects when analysing electromagnetic phenomena leads to the "loss" of physical phenomena in theoretical understanding [15, 16, 22].
6. The physical causes that can lead to the appearance of volume charge in a metal conductor with direct current were analysed.
7. It was shown that volume charge in a conductor with direct current can appear due to the pinch effect and as a result of relativistic changes in the charge density of conduction electrons and crystal lattice ions during their motion relative to an arbitrary reference frame.
8. An analysis was conducted of the neutrality conditions for a conductor with direct current, which are used in educational and methodological literature on electrodynamics [6, 26, 43]. Their contradictory nature and incorrectness within the framework of the generally accepted model of a conductor with direct current were demonstrated.
9. As a result of analysing the electromagnetic field created by a conductor with direct current in the traditional and in our proposed model, it was shown that when using the traditional model of a conductor with direct current, *the possibility of physical adaptation is violated, which "is a decisive condition for the implementation of physical relativity"* [8, pp. 8–9].
10. A consistent condition for the neutrality of a conductor with direct current was proposed [13]:
$$\rho_+^0 = -\rho_-^0,$$
according to which a stationary conductor with current is already characterised in its own reference frame by a volume charge density (19), [13, 15]:
$$\rho = \rho_+^0 - \frac{\rho_-^0}{\sqrt{1-\beta^2}} = \frac{\rho_0}{\sqrt{1-\beta^2}}\left(\sqrt{1-\beta^2}-1\right).$$
11. For methodological purposes, a mutually consistent substantiation of the transformation formulas for volume charge and current densities when transitioning from reference frame K to reference frame K' using the transformation formulas for 4-current components and based on the physical meaning of these quantities was presented, in the general case when $V \neq v$, and within the framework of each of the neutrality conditions for a conductor with direct current: $\rho_+^0 = -\frac{\rho_-^0}{\sqrt{1-\beta^2}}$ and $\rho_+^0 = -\rho_-^0$.
12. Within the framework of problem-based and task-based approaches, it was shown, under the neutrality condition for a conductor with direct current (19), that the force of interaction between parallel conductors with direct current, per unit length of each of them, with similarly and oppositely directed currents, respectively, equals:
$$F_y = \frac{\mu_0 I^2}{2\pi a}\cdot\left\{2\left(1+\frac{\sqrt{1-\beta^2}-1}{\beta^2}\right)\right\};$$

$$F_y = \frac{\mu_0 I^2}{2\pi a} \cdot \left\{ \frac{2}{\beta^2} \left(1 - \sqrt{1-\beta^2} \right) \right\}.$$

In the context of the problem-based approach, various manifestations of electromagnetic interaction and different descriptions of electromagnetic interaction between two conductors with direct current in a specific example are presented. This illustrates the principle of relativity, the methodology of its use, shows the invariance of the magnitude of the force of this interaction, confirms the reliability of the obtained result and its difference from the classical formula.

13. It was shown that when transitioning from one reference frame to another, the expressions for the force of interaction between two parallel conductors with direct current, taking into account the relativistic "charge of a conductor with current" and in the traditional model, where a conductor with current is neutral in its own reference frame, transform according to the requirements of relativity theory.
14. The invariance of force when transitioning from one reference frame to another within the framework of the traditional model and the model of conductors with current proposed by us in [12, 13, 15], and different magnitudes of this force, are determined by the same law of transformation of 4-current components and the different magnitude of the fourth component of 4-current in each of the models.
15. Based on the didactic principles of scientific and methodological orientation in studying electrodynamics, consistency, connection of practical experience with scientific provisions, we manage to illustrate: the effectiveness of the principle of relativity, the principle of correspondence, formulate the problem of choosing an adequate model, the dialectic of empirical and theoretical in the structure of physical knowledge and understanding the connection between theory and experiment, and thus coordinate the methodology of teaching electrodynamics with the methodology of the basic science to form students' worldview and physical style of thinking, ideas about methods of scientific cognition.

Declaration on generative AI: The author have not employed any generative AI tools.

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