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Elements of computer modelling: 12 studies for beginners. Study 1: Models and modelling

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Abstract. This paper presents the foundational concepts of computer modelling as both a cognitive method and educational approach, drawing from the pedagogical principles developed over decades of teaching practice. We examine the theoretical underpinnings of modelling as a scientific method, beginning with a conceptual framework for understanding models as simplified representations of reality that maintain essential characteristics of original phenomena. The paper provides a systematic classification of models (material vs. ideal, physical vs. analogue, intuitive vs. information-based) with illustrative examples demonstrating their application across disciplines. We establish mathematical modelling as the bridge to computer modelling and argue for its introduction in educational contexts through accessible tools like spreadsheets. The pedagogical approach advocated progresses from intuitive examples to structured model development, emphasising the iterative cycle of model creation, verification, experimentation, and refinement. This framework supports the development of interdisciplinary thinking and modern scientific literacy, positioning computer modelling as a key component in contemporary education. The principles presented form the foundation for a broader pedagogical system aimed at developing students' modelling competencies through progressively complex studies across various knowledge domains.

Keywords: computer modelling, mathematical models, modelling theory, model classification, information models, educational technology, interdisciplinary education, scientific literacy, knowledge representation, computational thinking

Foreword by Serhiy O. Semerikov to the third edition of Illia Olexandrovych Teplytskyi's "Elements of computer modelling"

In memory of Illia Olexandrovych Teplytskyi

It is with deep sorrow and a sense of irreplaceable loss that I undertook the preparation of this third edition of the textbook "Elements of computer modelling". Its author, Illia Olexandrovych Teplytskyi, was not only an outstanding scientist and educator who dedicated his life to the development of physics, computer science, and teaching methodologies in Ukraine but also my close friend and long-time colleague. His premature departure has been a profound loss for the entire academic community, especially for Kryvyi Rih State Pedagogical University (KSPU), to which he devoted a significant part of his scientific and pedagogical career, as well as for me personally.

This edition is published posthumously, and its primary aim is to preserve and make accessible the valuable scientific and pedagogical legacy of Illia Olexandrovych to a new generation of students, postgraduates, and lecturers. Computer modelling, the subject of this book, is increasingly gaining significance today, becoming not merely a research tool but a fundamental method of cognition in science, engineering, and education. Illia Olexandrovych was among those who deeply understood this trend and actively worked on incorporating modelling into the educational process.

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Our collaboration with Illia Olexandrovych spanned more than twenty years and encompassed a wide range of issues, from theory and methodology of teaching computer science and physics to the development of specific models and the use of modern software tools [4]. We worked together, wrote dissertations, published articles, participated in and organised conferences, and developed teaching materials, united by a common vision of the necessity to modernise science, mathematics, and computer science education in Ukraine. Computer modelling has always been at the centre of our joint interests, and we regard it as a key element in the training of modern specialists. Therefore, working on this edition is not only a professional duty but also a tribute to a person whose ideas and enthusiasm had an enormous influence on me, a friend who guided and supported me in every way. As the editor (and translator for a wider audience) of this edition, I have endeavoured to preserve as accurately as possible the author's original conception, structure, and unique pedagogical approach, presenting his work with profound respect for his intellectual heritage.

The author's scientific journey: Illia Olexandrovych Teplytskyi – pioneer of educational informatics

From 2000, Illia Olexandrovych Teplytskyi firmly linked his scientific and pedagogical activities with Kryvyi Rih State Pedagogical University (for more details about his journey, see [7]). His scientific interests lay in the theory and methodology of teaching computer science and physics [10], but the main work of his life became the research and implementation of computer modelling in the educational process.

The very topic of his PhD dissertation – "Development of creative abilities of schoolchildren by means of computer modelling" [8] – testifies to the author's innovative approach. For Illia Olexandrovych, computer modelling was not just a set of technical methods or a visualisation tool [3], but a powerful *method of scientific cognition* and, most importantly, an effective means of *developing creative thinking*, cognitive activity, and independence in students. He rightly believed that in an information society, it is insufficient merely to know how to program or use a computer; one must be able to construct and analyse models of real phenomena and processes to understand the world more deeply.

In his works, including this textbook, Illia Olexandrovych consistently advanced the idea that computer modelling could become the foundation for interdisciplinary integration, uniting knowledge from physics, mathematics, computer science, and other fields. He saw modelling as a tool for forming a scientific worldview, allowing various phenomena to be examined from a unified perspective. This view of modelling as a fundamental component of modern education, contributing to the development of higher-order cognitive abilities, runs as a common thread through all his scholarly work and finds vivid expression in the pages of this book.

The history and significance of the book "Elements of computer modelling"

The textbook "Elements of computer modelling", first published in 2005, was the natural culmination of many years of research and pedagogical practice by Illia Olexandrovych Teplytskyi in both school and university. The book evolved from the early 1990s, reflecting both the deepening of the author's understanding of the subject and the evolution of computer modelling as an academic discipline.

The second edition, published in 2010, was significantly revised and expanded [9]. Its inclusion in the KSPU jubilee series emphasises the recognition of the importance of this work for the development of computer science teaching methodology at the university. The publication of individual chapters of the textbook in leading Ukrainian scientific and methodological journals also attests to its relevance and demand within the pedagogical community.

The content of the book is constructed on principles of active learning. The author introduces the reader to the *technology* of creating a computer model and working with it, beginning with intuitively understandable examples that do not require specialised knowledge (such as the model of rumour propagation) and gradually progressing to more complex problems. The book examines diverse examples from physics (the flight of a paper aeroplane, the movement of bodies under the influence of gravitational force), biology (population dynamics), and other fields, often using accessible tools such as spreadsheets for modelling. This approach allows the reader not only to grasp theoretical foundations but also to acquire practical skills, learn to experiment with models, analyse results, and gradually refine the model to achieve greater correspondence with reality. The key concept of "model" is examined from multiple perspectives – both as a tool for cognition and as a means of reflecting reality.

"Elements of computer modelling" became one of the first and most significant Ukrainian-language textbooks in this field. The book is widely cited in scientific and methodological works and is used in the training of future teachers and specialists in information technology. Its importance for the Ukrainian educational system is difficult to overestimate, as it not only provides necessary tools but also shapes in its readers a modern, model-oriented style of thinking.

About the third edition

The need to prepare a third edition of Illia Olexandrovych Teplytskyi's book is driven by several reasons. First, the undiminished relevance of the subject itself – computer modelling – and the unique authorial approach to its exposition, oriented towards the development of thinking and creative abilities. Second, there is a continuing need for quality Ukrainian-language educational literature in this discipline. Third, we have a deep desire to preserve the memory of Illia Olexandrovych by making his major work accessible to new generations.

Working on this edition, I set myself the task of treating the original with the utmost care. My role as editor was primarily to preserve the author's structure, logic of exposition, didactic principles, and that special style which distinguished Illia Olexandrovych. The main goal was to ensure the accuracy and accessibility of his ideas.

This edition is based on the author's original, which was prepared in Ukrainian and Russian in 2014-2016 and revised by the author until his premature passing. Despite the title "Elements of computer modelling: 12 studies for beginners", until March 2018, we discussed, planned, and partially wrote a 13th study on neural network modelling [2, 5].

Since the book is published posthumously, only the most necessary changes have been made: certain references to software or external resources that may have become outdated since the second edition in 2010 have been clarified. The bibliography has been slightly refined and supplemented. However, all these corrections are technical and do not affect the essence of the author's exposition. When translating the book for a wider audience, the main task was to convey the meaning and style of the original accurately.

Addressing modern readers – pupils and teachers, students and lecturers – I would advise perceiving this book not merely as a guide to creating computer models. Try to see behind the specific examples and methods of the philosophy that the author invested in: the philosophy of modelling as a way of thinking, as a tool for deep understanding of the world and developing one's own intellectual potential.

The legacy of I. O. Teplytskyi and a look to the future

The scientific and pedagogical legacy of Illia Olexandrovych Teplytskyi has significantly enriched the theory and methodology of teaching computer science and physics in Ukraine. His innovative contribution consists primarily in the consistent development and implementation of computer modelling not as a narrowly specialised discipline but as a *key pedagogical strategy* aimed at forming in pupils and students systemic thinking, research skills, and creative abilities. His works, conference presentations, and many years of teaching at KSPU have had a notable influence on the development of educational informatisation in the region and country.

The book "Elements of computer modelling" occupies a central place in this legacy. It is not a simple collection of algorithms or descriptions of software tools. It is a deeply considered textbook reflecting the author's holistic view of the role of modelling in education and scientific cognition. The emphasis on fundamental principles, interdisciplinarity, and the development of cognitive skills makes this book relevant today despite the rapid development of information technologies. Its value lies not so much in describing specific tools (which may change) as in forming an *approach* to solving problems using models.

In concluding this foreword, I wish once more to express my deepest respect for Illia Olexandrovych Teplytskyi – scientist, teacher, colleague, and friend. This edition is dedicated to his bright memory. I hope that the book "Elements of computer modelling" will continue to serve as a source of knowledge and inspiration for all who aspire to master the art and science of computer modelling, thereby continuing the work to which Illia Olexandrovych devoted his life.

Figure 1: From left to right: Serhiy Oleksiiovych Semerikov, Valentyna Mykhailivna Teplytska, Vladimir Nikolaevich Soloviev, and Illia Oleksandrovych Teplytskyi photographed in September 2012.

Preface

This book emerged as the outcome of almost a decade of teaching the elective course "Computer modelling" to senior school pupils. The students' interest and positive reception of this course led to the idea that familiarity with computer modelling could prove interesting and beneficial to many other students as well.

Modelling has long been a powerful instrument of scientific cognition; it requires the integration of knowledge from different scientific disciplines and thus contributes to the formation of students' worldviews from the perspective of a unified – model-based – approach to studying various phenomena in the surrounding world: natural, technical, and social. Regarding the significance of model representations in science, the distinguished 20th-century physicist Max Born said: "All great discoveries in experimental physics have been due to the intuition of men who made free use of models, which were for them not products of the imagination but representatives of real things. How can an experimenter work at all, how can he communicate with his colleagues and contemporaries if he does not use models?" [6].

Today, modelling performs another important function – a humanistic one. It enables the prediction of negative consequences of human activity and helps prevent unwanted and dangerous phenomena, extending to global ecological catastrophes, including, above all, the threat of irreversible climate change or the terrible consequences of nuclear war, particularly a "nuclear winter". Due to these and similar reasons, computer modelling proves capable of influencing the content and style of political thinking in the modern world. It is part of scientific research work and belongs to those types of intellectual activity that can be mastered through studying specialised literature and analysing others' and one's own experience in such activities. The earlier such experience begins to be acquired, the higher the qualification of the future researcher may become. After all, researchers are not born – they become.

Computer modelling occupies a leading position in the practical application of electronic computing technology, so when discussing various practical applications of computers, we inevitably come to the necessity of introducing modelling to schoolchildren.

The main objectives of the proposed course are:

- acquaintance with the most widespread approaches to creating and researching computer models;
- study of simple methods for such work;
- formation of a culture of research activity using electronic computing technology.

The educational material includes a wide spectrum of problems from different subject areas and provides for the study of initial information about models and the technology of modelling:

- the entire modelling cycle is examined through concrete examples: problem statement → its content analysis → formalisation → model construction (informational, mathematical, computer) → algorithm composition → computational experiment (including verification of the model for adequacy) → interpretation of results → further model improvement;
- based on the nature of the phenomena being studied, deterministic and stochastic models are distinguished, and the peculiarities of working with models of each type are considered;
- such specific questions are discussed as the selection of an appropriate model type and appropriate environment for modelling, discretisation of processes to be processed on the computer, use of numerical methods, the origin of rounding

errors and ways to reduce them, initial information about model stability and some simple techniques for ensuring it;
- elements of a systems approach are implemented thanks to the possibility of creating different models for studying the same object and using the same models to research different objects.

The initially simplified description of the studied phenomenon is subsequently deepened: almost every model has at least three versions.

The conceptual apparatus is gradually accumulated, and the mastery of new working methods continues simultaneously, though the number of special concepts and terms is kept to a minimum. The material in the manual is subordinated to the requirement of *adequate use of the computer* in solving scientific and applied problems.

A computational experiment with a mathematical model eliminates many difficulties that arise in the analytical solution of a number of complex problems. This makes such problems quite accessible to senior schoolchildren, which in turn creates prerequisites for expanding the content of various educational subjects, as it becomes possible to include new research problems in them. We constantly draw students' attention to the appealing fact that *a successfully created model is capable of helping the researcher obtain new or additional information about the object of study.*

Practical work with computer models presupposes resolving the problem of choosing an environment for modelling. Initially, spreadsheets proved to be quite a suitable environment, the study of which is provided for by the current programme of the school course in computer science. However, with all their simplicity and clarity, spreadsheets are effective only for relatively simple and (or) specially selected problems. However, they do allow one to gain a basic understanding of the peculiarities of computer modelling technology. In general, when studying modelling, one cannot limit oneself to just one environment. As knowledge of computer science is acquired, it is rational to transition from one environment to another, more sophisticated or better adapted to a specific task. The use of spreadsheets allows for the systematic study of computer modelling to begin much earlier than the study of programming languages.

The manual consists of 12 chapter-studies, arranged in five sections and not connected by any plot.

In the first section (chapters 1 and 2), general ideas about models and modelling are formed and subsequently clarified; an example of one of the possible classifications of models is provided. Special attention here is devoted to mathematical computer models, as they constitute the main content of the course. The two examples presented here illustrate the capabilities of the computer as an effective computational device and convenient tool for constructing graphs of mathematical dependencies.

The second section (chapter 3) introduces the basics of the technology for creating a model and subsequent work with it. The plot basis of the very first model is, in general, an outline that is well understood and does not require specialised knowledge. This is, at first glance, a simple but, in reality, not at all trivial and, moreover, multifaceted problem about the spread of rumours. The first results of working with a simplified version of this model reveal its almost complete qualitative discrepancy with facts known from life observations. In subsequent versions of the model, it is gradually complicated by introducing new significant factors into consideration. As a result of such actions in sequence, the model becomes increasingly adequate, and the results of its study are more reliable. In this process, we adhere to the principle of correspondence, according to which each next improved version should contain the previous one as a special case.

The third section (chapters 4–6) is devoted to considering several classical problems of mathematical ecology. Chapter 4 examines four versions of a single-species population model. First, a simple model of population dynamics in the absence of limitations (Malthus model) is investigated and, as its development, a population model with limitations related to competition for environmental resources (Verhulst-Pearl model). Its logical continuations turn out to be two models of industrial use of the population. The first is built on the basis of a rigid plan for removing biomass, and this leads to the appearance of unstable equilibrium in numbers and the danger of irreversible extinction of the population. In the second model, flexible regulation (self-regulation) is implemented on the basis of negative feedback, which reliably ensures stable equilibrium states. The interpretation of these results for the last two versions also proves useful for the formation of elements of ecological culture in students.

In the two subsequent chapters, the model of coexistence of two-species populations based on "predator-pre" relationships (Volterra-Lotka model) and the age model of a single-species population (discrete matrix model of P. Leslie) are considered, respectively. Comparing modelling results using the Malthus and Leslie models shows their qualitative similarity.

The problems proposed in the fourth section of the manual (chapters 7-10), unlike the previous ones, require knowledge from the physics course. Examples of mechanical movements under the action of variable forces are proposed for investigation here, specifically those that depend on coordinates (elastic force and gravitational force) and velocity (resistance force and aerodynamic force). The main feature that distinguishes these problems from most school problems in mechanics is that under the action of variable forces, bodies acquire variable accelerations, and the exact calculation of their coordinates at an arbitrary moment in time becomes impossible using methods of elementary mathematics. Thus, prerequisites for introducing students to simple numerical methods arise naturally.

The first object of research in this chapter is the oscillatory motion of a body on a spring, taking into account resistance (chapter 7). Testing and debugging of the model is conducted only in the case of elastic force acting on the body since the patterns of such motion are well-known from the school physics course. Of interest to us here is the discussion of the issue of the accumulation of computational errors and improving the accuracy of calculations by improving the algorithm. Taking into account resistance forces gives pictures of damped oscillations, which noticeably differ when transitioning from viscous friction to dry friction.

The next object is the motion of a body in a gravitational field (chapter 8). Four problems are proposed here:

1) the motion of an artificial satellite around a planet (or the motion of a planet around the Sun);
2) the motion of a natural satellite of a planet (as well as components in "binary star" systems), where there is a need to transition to a reference system associated with the common centre of mass;
3) quantitative verification of the laws of planetary motion (Kepler's laws);
4) study of the question about the appearance of planetary orbits when transitioning from a heliocentric reference system to a geocentric one.

Chapter 9 examines the problem of a paper aeroplane's flight under the action of gravitational and aerodynamic forces, which manifest themselves through two components: drag and lift forces. Since each of these forces, according to Zhukovsky's theorem, is proportional to the square of velocity, in addition to the question about the shape of the trajectory of motion, the subject of special discussion here is the search for numerical values of the corresponding proportionality coefficients for Zhukovsky's theorem (model coefficients).

The fourth section concludes with the development of a game problem about a soft landing on the Moon (chapter 10) with elements of landing mode optimisation and the creation of a user-friendly interface.

Chapters 11 and 12 are devoted to studying stochastic models through familiarisation with the random sampling method (Monte Carlo method). Here, the model of Brownian motion and a problem in operations research are considered. The construction of the first model introduces the world of random variables and mathematical statistics. It contributes to the formation of ideas about probability distributions, in particular, illustrating two common distributions: uniform and normal. The research of the second model shows that in some situations, it is possible to significantly reduce the number and cost of repairs of technical equipment solely by choosing the optimal organisation of service. In addition, the opportunity arises to understand how it becomes possible to model real random events with the help of uniformly distributed random numbers and the known law of distribution of relative frequencies of equipment failures, thus evaluating the effectiveness of the Monte Carlo method. No less important here is the conclusion that the research of such models often allows separating potentially productive decisions from erroneous ones.

Since the proposed course is elective, working with this manual does not presuppose mandatory consideration of all the examples presented without exception. For the same reason, similar reasoning is provided in a number of tasks, and the teacher can select educational material at their discretion.

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Critical remarks will be accepted with gratitude at the address Ukraine, 50086, Kryvyi Rih, Universytetskyi Avenue, 54, Kryvyi Rih State Pedagogical University, Department of Computer Science and Applied Mathematics.

Étude – French *etude* – *study* (literally)

An exercise that serves for the development and perfection of the technique of some art: acting, chess, musical, etc., usually of an improvisational character.

An exercise that serves for the development and perfection of the technique of the art of modelling.

The Author

Section I. Modelling as a method of cognition

The word "model" derives from the Latin modulus (measure, pattern, norm). The initial meaning of the word was associated with crafts – foundry work, construction, and so on.

Ancient Roman foundrymen called the samples of future castings models; in the medieval era, this word meant the scale in which all proportions of a building were presented, and later, it acquired the meaning of a pattern according to which something is created in art. Since the pattern was often a reduced copy of the object (original), the notion of a model as a miniature representation that resembled the object in everything except size arose. In later times, the concept of a model acquired a general scientific character, and it began to be used in scientific research, particularly in those cases when direct research proved impossible or ineffective. In such situations, the studied object-original, due to its significant complexity, was replaced by another – imagined or artificially created, simpler and more accessible for study, and one that was in some correspondence with the original. This *substitute object was called a model of the object being studied.*

Thus, the creation and research of models (modelling) as one of the basic tools of world cognition has been known for a long time. It appeared simultaneously with scientific cognition and always actively accompanied it. Moreover, scientific cognition in its current understanding without modelling (also in its current understanding) is generally impossible.

Human learning is also cognition, but this cognition differs from scientific cognition in that, when learning, a person, with the help of teachers, books, and modern information and communication technologies, learns already-known facts in a relatively short time, whereas science learns what is new and unknown over a much longer time. However, both scientific and educational cognition have common features, which include working with models, that is, modelling.

The human mind is incapable of simultaneously encompassing all the complex interactions and mutual influences of an object and its surroundings. It is for this reason that cognition was divided into separate areas – sciences. However, the foundations of sciences are unified. "Science is an internally unified whole, and its division into separate areas is conditioned not so much by the nature of things as by the limitations of human cognitive capabilities. In reality, there exists a continuous chain from physics and chemistry through biology and anthropology to the social sciences... The great internal similarity is also possessed by research methods...". At first glance, it might seem that the author of these words is a representative of "humanitarian thinking"; in reality, these words belong to the outstanding German physicist and Nobel Prize laureate Max Planck.

In attempting to understand (research, study) an object, a researcher is forced to resort to simplifications and limitations. Moreover, within each separate science, cognition usually proceeds through simplifications and limitations and further – from the simple to the complex.

The development of any science has long been impossible without the creation of *theoretical models – theories, laws, hypotheses* – that reflect the structure, properties, and behaviour of real objects. The creation of new theoretical models significantly changes people's understanding of the surrounding world. This happened after Nicolaus Copernicus created the heliocentric system of the world, after James Watson, Francis Crick, and Sydney Brenner developed the model of the double helix of the DNA molecule, after the creation of the human genome model, and so on. For example, speaking about the structure of particles of matter – molecules – we, as our knowledge about them is enriched, initially depict them as densely arranged coloured circles – atoms – in one plane, later as chains of atoms on a plane, and then in three-dimensional space with depictions of angles and distances between atoms. All such representations are models. These models are not static: each of them only at the moment of creation (or study) reflects existing information about the object and always remains open to further clarifications and improvements.

We do not always realise the following fact: every day and every hour, we deal with models. Rather the opposite, in our understanding of the role that modelling plays in our own lives, we often resemble the famous Molière character Jourdain (from the play "The Bourgeois Gentleman"), who unexpectedly discovered one day that for forty years he had been speaking prose. Modelling in our life is everyday prose, but someday, the moment of realisation of this fact must come. The author very much wants you, dear reader, to know that such an enlightenment would come earlier than at the age of forty. In addition, the author aims to draw your attention to one of the most effective modern technologies of cognition, the name of which is Computer Mathematical Modelling.

You will be able to familiarise yourself with some ideas and methods of this technology and apply them to the creation and experimental research of computer mathematical models of many interesting and diverse phenomena and processes. Moreover, for those of you who are familiar with programming, broad opportunities open up for the creative implementation of your own ideas.

Goethe justly noted: "What a person does not understand, he does not possess" [1, p. 78]. That is why the first step on the path to mastering the art (technology) of computer modelling is overcoming the problem of understanding.

1. Models and modelling

For several centuries now, the concept of "model" has been widely and variously used in different branches of natural science and technology, in the humanities and fine arts, in economics and psychology, in designing clothes, footwear, hairstyles, and in literature. That is why giving an immediately understandable description or, moreover, providing such a definition for this concept that would correspond to its various applications and at the same time be accessible to specialists in different fields, especially to schoolchildren, is not at all simple and, most likely, impossible. In such cases, it is better to turn to examples.

1. Wishing to explain how a radio receiver works, one often refers to a drawing where, with the help of conventional symbols, its individual elements and their combinations (blocks) are depicted, and arrows between them show the directions

along which electrical signals move. The image on paper, which is called a *schematic*, is a model.

2. In preparation for implementing a biological method of pest control for agricultural crops, simplified copies of ecological systems are created in special laboratories, where representatives of cultivated plants, their pests, and also enemies of these pests are placed. The peculiarities of interaction among all representatives of the specified community are studied in such laboratory systems. The laboratory ecosystem is a model.
3. Before starting serial production of a new type of aircraft (ship, automobile), first its reduced copy, and subsequently, if possible, the object itself is placed in a wind tunnel, and with the help of special equipment, it is determined what loads arise in different elements of the construction. An aircraft in a wind tunnel is a model.
4. In order to estimate the minimum length l of wire that needs to be stretched from the roof of a five-storey building to the roof of a nine-storey building located nearby, one can measure the distance d between the buildings and the difference in heights of these buildings h, and then calculate

$$l = \sqrt{d^2 + h^2}$$

The given formula, when used in the conditions of this specific problem, is a model.

It is clear that becoming acquainted with the operation of a radio receiver is impossible, even when looking inside it, so the only thing that remains here is the schematic.

Understandably, one could acquire a certain number of enemies of pests and immediately settle them in the region. But it might turn out that they would also become enemies of some beneficial insects or the timing of their mass emergence would not coincide with the corresponding timings of the pests. Therefore, firstly, the expenses associated with their acquisition and maintenance would not be justified, and secondly, a significant part of the harvest would perish again. It is better to perform laboratory research because the enemies of our enemies may not turn out to be our friends.

Of course, one could launch an aircraft into serial production without having an accurate idea about the loads that would arise in its individual constructions under flight conditions. However, if these loads prove significant, they would destroy the aircraft in flight. It is better to study the aircraft in a wind tunnel.

One could, without performing any calculations, take turns lifting the wire to both roofs, unwinding it from a reel located on the ground. But in such a case, one would have to lift a wire of noticeably greater length than needed to the second roof, and, consequently, of greater mass. In the case that this is a heavy cable, it is better to spend some time on a preliminary, uncomplicated calculation and cut the wire more precisely.

In all the named examples, there is a comparison of some real object-original with another that replaces it:

- radio receiver – schematic on a sheet of paper;
- ecosystem of a region – ecosystem in a laboratory;
- serial aircraft – singular aircraft in a tunnel;
- approximate length of wire – formula.

In each of the examples, it is assumed that some properties are either preserved in the transition from the real object being studied to its substitute – the model – or, at the very least, the model allows the forming of at least some ideas about such properties. Thus, the conventional symbols and arrows on the schematic have nothing in common with the individual elements – radio parts and their groups – in the radio receiver, but only the schematic allows an understanding of where electrical signals go to and from and what changes occur with them.

An aircraft fixed in a wind tunnel does not fly, yet the loads that arise in its individual elements correspond to real flight conditions.

The variables in the formula notation do not resemble either the Earth's surface or buildings, yet the given formula allows saving wire and accomplishing the planned work in less time and with less effort.

Now let us consider the following definitions:

A model is a material or mentally imaginable object which in the process of research (cognition) *substitutes* for the real object of study – the *object-original* – and at the same time has some important features of the original for this research.

Here, the term "object" is used in the broadest sense: an object can be not only a certain thing but also any situation, phenomenon, process, etc. A model is like an intermediary between the object being researched and the researcher.

Modelling is a form of research activity when, in order to obtain new information about an object, not the object itself but its simplified model is experimentally investigated.

An appropriate comparison here is that a researcher who creates a model is similar to a caricaturist, who, as is known, does not reproduce the original in all details, as a photographer does but simplifies it so as to reveal and emphasise the most characteristic features.

A good model of an object should be precisely a good "caricature" of it, which reflects the most characteristic and typical signs and properties of the original and deliberately ignores all others – non-essential properties.

Any scientific research – both theoretical and experimental – is based on modelling since a model is always more accessible for study than the real object.

It should be understood that there are objects that cannot be researched directly at all. For example, cognitive experiments with a country's economy are unacceptable; field experiments with the past, with stars or stellar systems – galaxies – are fundamentally unfeasible; it is impossible to research processes with very long or extremely short durations in time, and so on. The only method of research in such cases is the method of modelling.

A model also allows learning how to effectively manage an object by testing various management options on the model of this object. Researching the object itself for this purpose, in the best case, is inconvenient and often simply dangerous or impossible since there is a threat of bringing the object into an undesirable or irreversible emergency state. A very sad example here is the accident at the Chornobyl Nuclear Power Plant in 1986. If an error is detected and corrected on a model, it is always much simpler, safer, and cheaper. Another convincing example is the failure of the power supply system of the spacecraft Apollo 13 during its stay in the Moon's orbit (in 1970). Only the recreation of the emergency situation and the playing out of options for its elimination on a model at the flight control centre allowed the spacecraft with astronauts to return to Earth.

The most attractive fact in modelling is that a successfully created model has a strange property: it is capable of providing new information about the object-original.

In cases where an object has characteristics dependent on time (so-called *dynamic* characteristics), the task of predicting changes in the states of the object acquires special significance. Here, the use of models also provides great assistance.

Thus, modelling enables:

- understanding the structure of a specific object, its basic properties, patterns of development, and interaction with the environment;
- learning to manage the object in order to identify the best methods (regimes) of management according to a given purpose;
- predicting the consequences of specified methods of influence on the object.

1.1. Classification of models (their types)

Here, we will discuss physical and analogue modelling, as well as consider intuitive, informational, and sign models.

It is clear that models can differ greatly in many properties, so there are certain approaches to dividing them into separate groups according to some common features. First of all, these are two large groups – **material** (physical, substance) and **ideal** (mental, imaginary) models. In turn, they are divided into more specific types.

In material modelling, for example, **physical** and **analogue** modelling are distinguished.

Physical modelling

Physical modelling refers to modelling in which, for research purposes, a real object is compared with its enlarged or reduced copy (usually in laboratory conditions). Results obtained on the model are transferred to the object in accordance with special rules. The basis for such a possibility is the theory of similarity developed by Isaac Newton, which is the *theoretical foundation of physical modelling.*

Let us provide several examples of physical models:

- in hydraulic engineering – troughs with water that model rivers, canals, locks;
- in construction – models of structures on which stability conditions and load distribution are studied;
- in aircraft and shipbuilding – reduced copies of aircraft and ships.

Physical models can also be simply illustrative:

- in physical geography – a tellurium that models the daily and annual rotation of the Earth and related consequences (change of day and night, change of seasons, eclipses, existence of climate zones, etc.);
- in astronomy – a model of the celestial sphere and a planetarium that models the rotation of the starry sky visible from Earth and the change in its appearance during the day or with the change of seasons;
- in thermal engineering – models of thermal engines of cyclic operation principle (steam engine, internal or external combustion engine), engines of continuous operation (steam and gas turbine), jet (including rocket) engines.

Example of the use of similarity theory:
models of ship movement in test channels

Physical phenomena can be characterised not only by quantities that have a certain dimension but also by dimensionless combinations of such quantities. These dimensionless combinations, composed according to certain rules, are called *similarity criteria.*

For two physical processes to be considered similar, it is necessary and sufficient that they be qualitatively identical and their similarity criteria be pairwise equal.

Then, knowing the values of quantities (for example, velocity, pressure, temperature, density, etc.) that characterise one object (model), one can find the value of one of the corresponding quantities that characterise the other (object-original). This is precisely how, based on the results of research conducted on the model, the corresponding parameters of the original are determined.

Different groups of phenomena have their own similarity criteria. Each of the criteria bears the name of the scientist who first proposed it. In particular, one of the basic similarity criteria in hydrodynamics (which studies the movements of bodies in liquid media) is the Froude number (named after the 19th-century English naval scientist William Froude):

$$Fr = \frac{v^2}{gl} \tag{1}$$

where v is the velocity of the liquid at a significant distance from the body it flows around;

l is the characteristic linear dimension of the body;

g is the acceleration due to gravity.

The Froude criterion plays an important role in modelling processes related to ship movements. At low speeds (for slow-moving vessels), the Froude criterion $Fr = 0.4$ under the condition that the speed is determined in knots (nautical miles per hour).

Before beginning the construction of a new ship, its reduced copy – a physical model – is studied in a test channel. One of the many questions that interest shipbuilders is the question of speed. With the ship model, everything seems clear: it is significantly reduced in comparison with the actual vessel. Thus, a model of a two-hundred-metre tanker, made at a scale of 1:200, has a length of 1 m.

Let us suppose that the real tanker should develop a speed of 19 knots. Let us pose the question: at what speed should the model move in the channel so that, based on the measurement results during its testing, calculations could be made for the real ship? Is it also at a speed of 19 knots, or perhaps the speed should be reduced to the ratio of 1:200?

Froude experimentally proved that the maximum speed of a vessel provided that no technical "tricks" are used, never exceeds the value:

$$v_{max} = 2.5\sqrt{\text{length along the waterline}}\ \text{knots} \tag{2}$$

The tanker's two-hundred-metre hull has a length along the waterline of about 190 m, and the model – correspondingly 0.95 m.

The maximum speeds of the tanker and the model, respectively, are:

$$v_{max\ t} = 2.5 \cdot \sqrt{190} = 34.5\ \text{knots} = 64\ \text{km/h}.$$
$$v_{max\ m} = 2.5 \cdot \sqrt{0.95} = 2.4\ \text{knots} = 4.5\ \text{km/h}.$$

Experiments with the model show that when it is pulled in water faster than 2.4 knots, it simply "jumps" out of the water. So, 2.4 knots is indeed the maximum possible speed of the model.

Thus, when testing a future two-hundred-metre tanker on a model, two laws of similarity should be kept in mind:

1) similarity of geometric dimensions of the model and the real vessel;
2) the similarity in modelling the speed of the vessel in the water.

The shipbuilder needs to know in advance how the tanker will behave not only at maximum speed but also at economical speed on a regular voyage. The answer to this question will be obtained from the law of similarity – equality of the Froude criteria for the tanker and the model:

$$\left[\frac{v^2}{gl}\right]_t = \left[\frac{v^2}{gl}\right]_m$$

And finally, expressing the speed of the real vessel (tanker) through the speed of its model, we have:

$$v_t = v_m\sqrt{\frac{l_t}{l_m}} = v_m \cdot \sqrt{\frac{190}{0.95}} \approx 14.1 \cdot v_m$$

Historians of science and technology consider Froude's works to be the first examples of scientifically substantiated practical use of the modelling method.

Note. Besides the Froude criterion, a number of other criteria are considered in hydroaerodynamics, among which is the widely known Mach number M. The Mach number is a quantity equal to the ratio of the velocity v of a gas flow to the speed of sound a at the same point in the flow: $M = v/a$. This is the basic similarity criterion for the flow of a compressible gas. It is named after the Austrian physicist and philosopher Ernst Mach.

Exercises

1. Show that the Froude criterion is a dimensionless combination of its components.
2. Establish the relationship between the units of speed *knot* and *km/h*.
3. Using the principle of relativity of motion, answer the question: what does the phrase "the speed of the aircraft equals $2M$" mean?

Analog modelling

Analogue modelling is based on analogy (external similarity in certain features) of processes and phenomena that have different physical natures but are described by similar-looking equations, logical schemes, etc., that is, by the same mathematical models. This allows for the research of some phenomena using models of completely different ones that are more accessible under certain specific conditions.

Thus, in particular, using the formula:

$$\frac{1}{Z} = \frac{1}{X} + \frac{1}{Y} \tag{3}$$

Where Z is the sought quantity, one can find:

- the total resistance $R = Z$ of two parallel connected conductors with resistances $R_1 = X$ and $R_2 = Y$;
- the focal length $F = Z$ of a thin lens, if the distance $d = X$ from the lens to a luminous point and the distance $f = Y$ from the lens to the image of this point are known;
- the time $t = Z$ of joint performance of some work by two performers under the condition that one of them is capable of performing all the work in time $t_1 = X$, and the second – in time $t_2 = Y$;
- the time $t = Z$ of a passenger's movement on a moving escalator, if this passenger covers the steps of a stationary escalator in time $t_1 = X$, and the escalator itself moves a stationary passenger in time $t_2 = Y$.

All this means that by constructing an electrical circuit with two variable resistors (for example, rheostats) connected in parallel, we will be able to solve any of the aforementioned problems. If only we could determine the resistance of each of the resistors and their total resistance. By the way, consider how this can be practically done, and at an opportunity, be sure(!) to conduct such an experiment in the physics room. Then, you will have the opportunity to understand the basic operating principle of *analogue computing machines* (ACMs), in which information processing occurs with the help of specially selected physical processes that model the studied pattern. As a rule, processes in electrical circuits (as in our example), pneumatic devices, etc., serve as such. Another feature of analogue modelling and ACMs is working not with discrete but with continuous (analogue) signals.

The most well-known examples of analogue modelling:

- studying mechanical oscillations using mathematical models of electrical circuits and vice versa;
- researching the movements of air masses using the same movements of liquids;
- studying the characteristics of the gravitational field using the corresponding characteristics of the electrostatic field.

We note that in both types of material modelling, models are some material embodiment of the object-original and are always connected with it by their geometric, physical, or other characteristics. In addition, the research itself – a full-scale experiment – is associated with material action on the model.

Fundamentally different from material modelling is **ideal** modelling, which is based not on the material analogy of the object and the model but on an ideal, imaginary analogy. It arises in human consciousness and exists within it. An ideal model can exist in the imagination of one individual person, a group of people, or society. Ideal modelling has a theoretical, abstract character. Two types of ideal modelling are considered: **intuitive** and **sign**.

Intuitive models

Intuitive models are based on an intuitive understanding of the object of research; they are not subject to mathematical description or do not require it. Images of various objects that arise in our consciousness are attributed to intuitive models of these objects.

The well-known children's game "What would happen if…?" is, in fact, a game of prophecies. Perhaps children like it due to their natural need to build models of behaviour for correct anticipation and prediction of future life situations. Even before coming to school, children already construct their own models in their heads (in imagination), with the help of Lego constructors, on paper, or on a tablet.

Examples of intuitive modelling are also well-known cases when there is a need to decide conditions of lack of information, such as choosing an optimal strategy for future actions. Often, groups of specialists (experts) are involved in solving such a problem, and one of the well-known methods of making collective decisions called *brainstorming*, is used. Brainstorming is an operative method of collective problem solving based on *stimulating creative activity*. Here, each participant in the discussion is encouraged to express as many solution options as possible, including improbable and fantastic ones. Further, from the total number of expressed ideas, the most successful ones that can be used in practice are selected.

In this regard, some psychologists tend to believe that making correct decisions in the mentioned situations is based on the ability of our brain to subconsciously supplement the lack of information from previous life experiences. That is why each person's life experience can be considered their own intuitive model of the surrounding world. However, when making responsible decisions, it is not worth exaggerating the role of human intuition and relying solely on it.

Information models

The technical models we have considered above are material, also called full-scale. In fact, such models are not the subject of study in computer science, but it was convenient for us to discuss them because examples of material models are more understandable and visual for beginners. Experience shows that after examining some general properties of models on such examples, it is easier to proceed to study the properties of information models.

Computer science studies information models. Examples include a model of a school class – the class register, a model of train movement at a station or aircraft at an airport – the train or flight schedule, a weather model – the hydrometeorological centre forecast, a model of educational process organisation – the lesson timetable. The characteristics of the listed objects are sets of quantities (data), the values of which can be numbers, images, texts, dates, etc. Characteristics of any objects – these and previously described – are commonly called *parameters*.

A model that describes an object by a set of parameters and connections between them is called informational. An information model is a certain description of the object of modelling; it is some information about this object, and as is known, information can be presented in different forms. That is why there are different forms of information models: verbal, mathematical, graphical, tabular, etc.

Experience in creating information models shows that the most challenging stage after determining the parameters is identifying the connections between them. Indeed, compiling a lesson timetable, among many tasks, is reduced to ensuring, for example, that two teachers do not appear at the same lesson in the same class, that mathematics lessons are not the first, that during the working day there are no "empty" lessons in any class, that the study load decreases by the end of the week, that a methodological day – a day for self-education and free from lessons – is provided for each teacher, and many, many different "thats". The complexity of such a task increases very quickly when attempting to take into account yet another property, yet another connection. That is why, to this day, there is no system of fully automated compilation of lesson timetables, and those programs that exist provide for joint work of the program and a human dispatcher, that is, a human-machine system.

However, one should not think that identifying connections between parameters is always so complex. In reality, a huge number of tasks are significantly less complex, and building information models for them proves to be quite accessible.

Concluding a brief discussion about intuitive and information models, let us once again note that intuitive models exist in human consciousness while information models exist in computer memory. That is, humans and computers use absolutely different models of the world. Human thinking is characterised by imagery, ambiguity, and approximation, whereas information entered into a computer must be concrete, accurate, and subject to rigid logic.

Sign models

The main type of information modelling is sign modelling, which uses models of sign transformations of some kind: schemes, graphs, drawings, formulas, and sets of symbols. Here are some examples:

- various schemes (electrical, kinematic, etc.);
- formulas (equations) of mathematics, physics, technology;
- records of chemical reactions;
- geographical maps;
- musical notations of melodies;
- various pictograms, etc.

Outside of human consciousness and activity, no sign system is possible.

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1.2. Mathematical modelling

An extremely important and widespread type of sign modelling is mathematical modelling, in which research is carried out using models formulated in the language of mathematics, that is, in the form of mathematical expressions.

Let us recall that information (data) about objects (events, processes) and connections between them is already some model of them, namely, an information model. A special case of information models is mathematical models.

A *mathematical model* is a description of some real process or some studied situation in the language of mathematical formulas and relations (equations, inequalities, logical expressions, etc.); it represents in mathematical form the basic patterns and connections inherent to the object being studied.

Let us recall the well-known problems from primary school on composing equations: "Two pedestrians (cars, trains) set out towards each other...", "Two pipes are installed in a pool..." In these examples, you, in most cases, easily composed the necessary equations without any talk about the fact that you were engaged in building mathematical models. In such problems, everything is so simple that the words "mathematical model" seem like inappropriate pathos. However, it is precisely here that the real process of abstract mathematical concepts is described in the language.

The formula with reciprocal quantities given above is a generalised mathematical model of the phenomena listed there. Thus, similar models often allow for studying different phenomena. However, no less important is the converse fact – the possibility and expediency of studying the same phenomenon using different models. In such a case, the results of modelling complement each other, giving a more complete understanding of the phenomenon being studied.

A mathematical model is always a simplified image of some real situation; it allows for reducing a complex problem to a comparatively simple mathematical problem.

Mathematical modelling has long been, since the 17th century from the time of Isaac Newton – its founder – successfully applied in various branches of science and technology. However, the widespread use of this method was constrained by imperfect technical means. But in the mid-20th century, mathematical modelling experienced its second birth. This happened thanks to the emergence of electronic computers, which provided specialists with the opportunity to work with complex mathematical models that contained thousands of various parameters and unknown quantities. The first electronic computers that appeared in the 1950s were created precisely for processing mathematical models related to mastering nuclear energy and developing and improving rocket and space technology.

1.3. Computer modelling

The involvement of computers in working with models gave rise to the modern technology of model research – computer modelling. A *computer model* is a mathematical model of a real process or phenomenon, implemented by computer means (modelling environment – specialised or created by means of some programming language).

To involve a computer in creating and studying a mathematical model, one should perform such a transformation of it so that it becomes "understandable" for the computer, or, as they say, "it is necessary to prepare the problem for solving on a computer". Since the model being transformed is mathematical, the modelling environment must provide the possibility of working with numerical data. One such environment is spreadsheets, which are well-known to you.

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